Algebra I Vocabulary Cards

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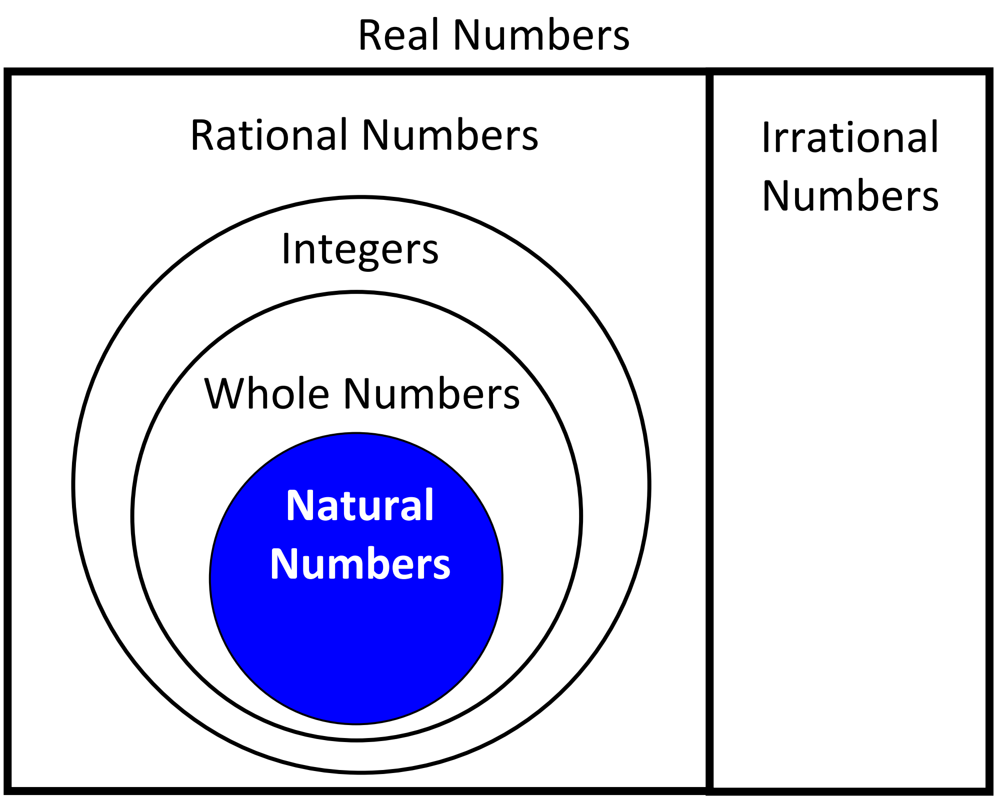
Relations and Functions

Statistics

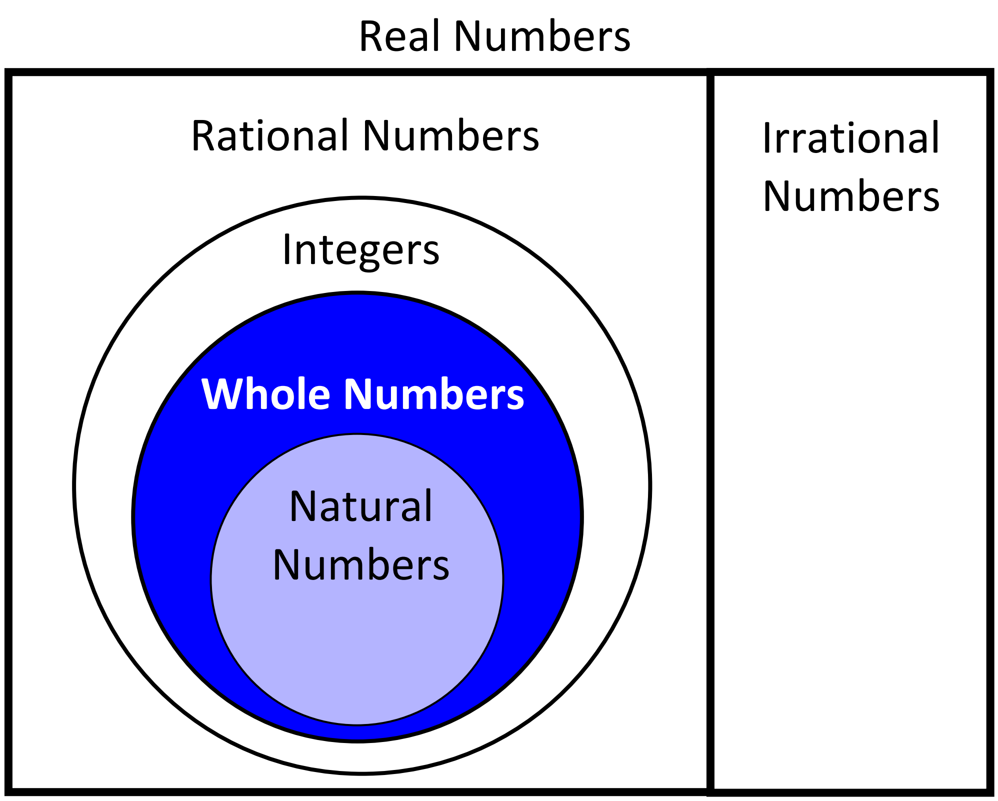
Natural Numbers

The set of numbers 1, 2, 3, 4…



Whole Numbers

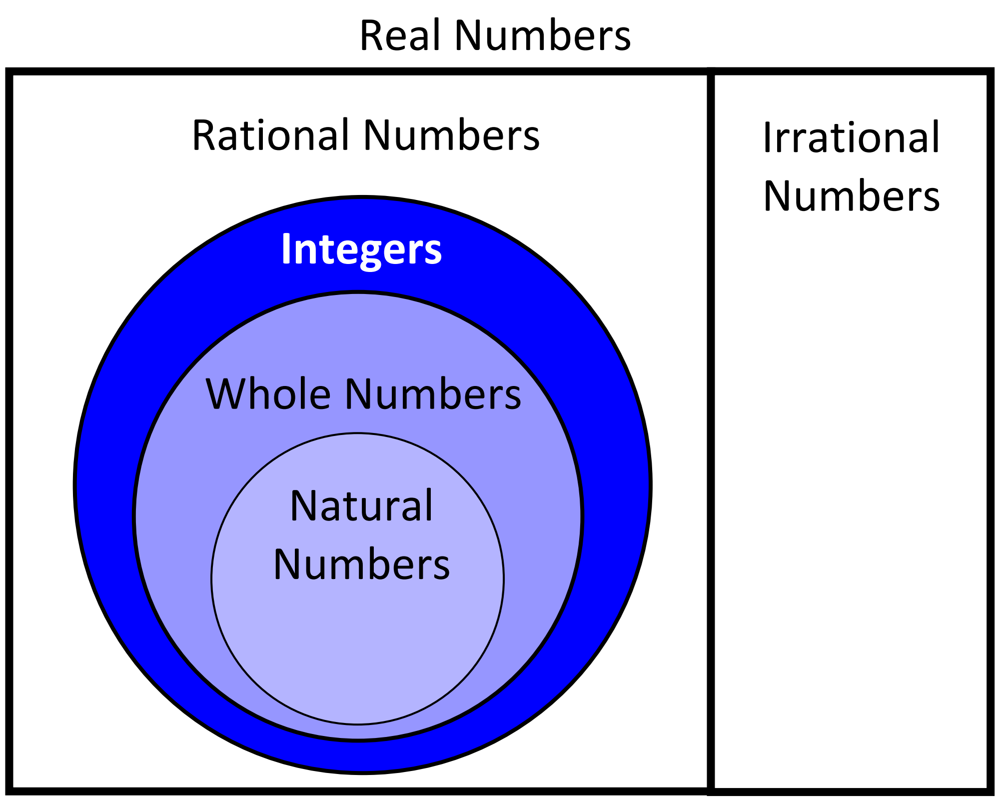
The set of numbers 0, 1, 2, 3, 4…



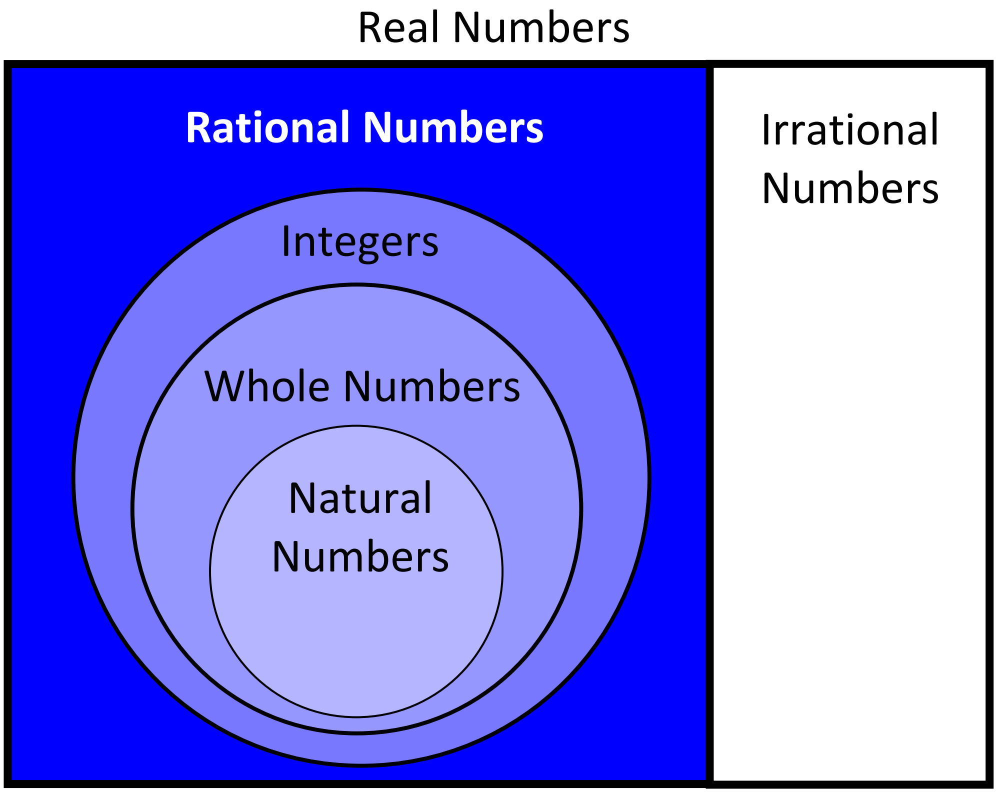
Integers

The set of numbers

…-3, -2, -1, 0, 1, 2, 3…



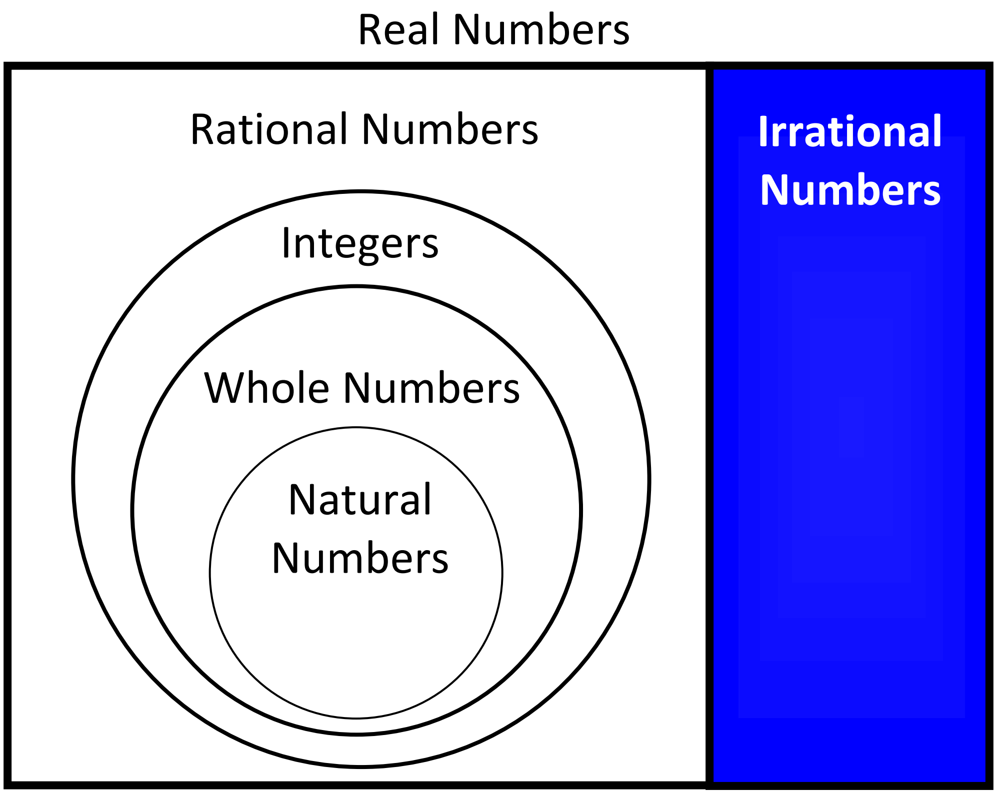
Rational Numbers



The set of all numbers that can be written as the ratio of two integers with a non-zero denominator

$$2\frac{3}{5}, \quad -5, \quad 0.3, \quad \sqrt{16}, \quad \frac{13}{7}$$

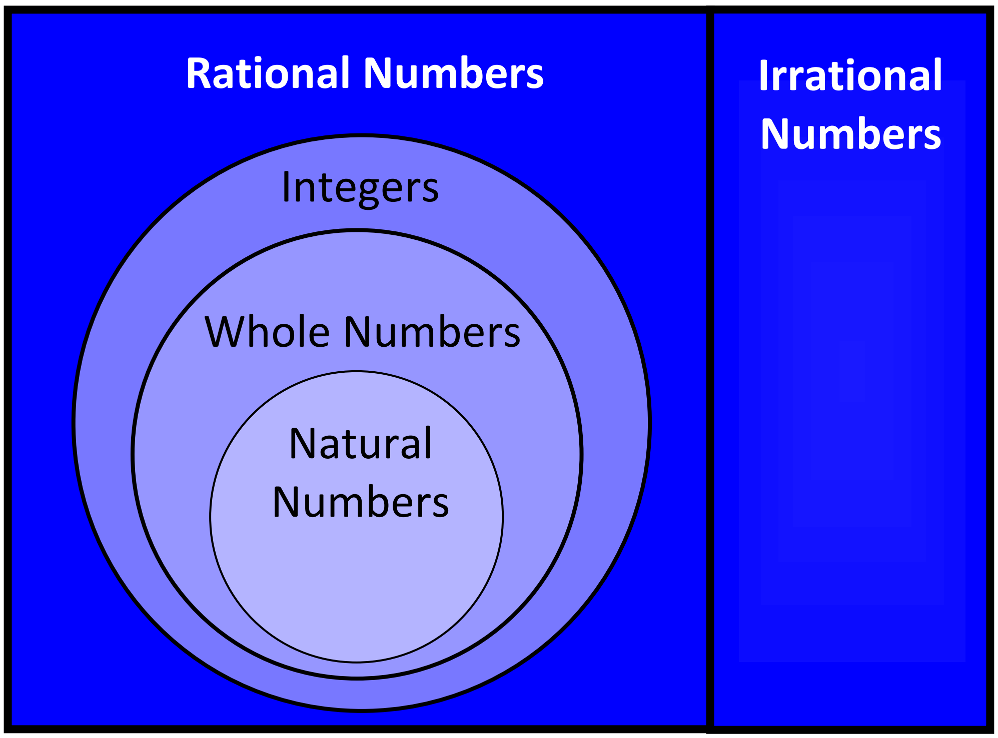
Irrational Numbers



The set of all numbers that cannot be expressed as the ratio of integers

$\sqrt{7}$, π , -0.23223222322223…

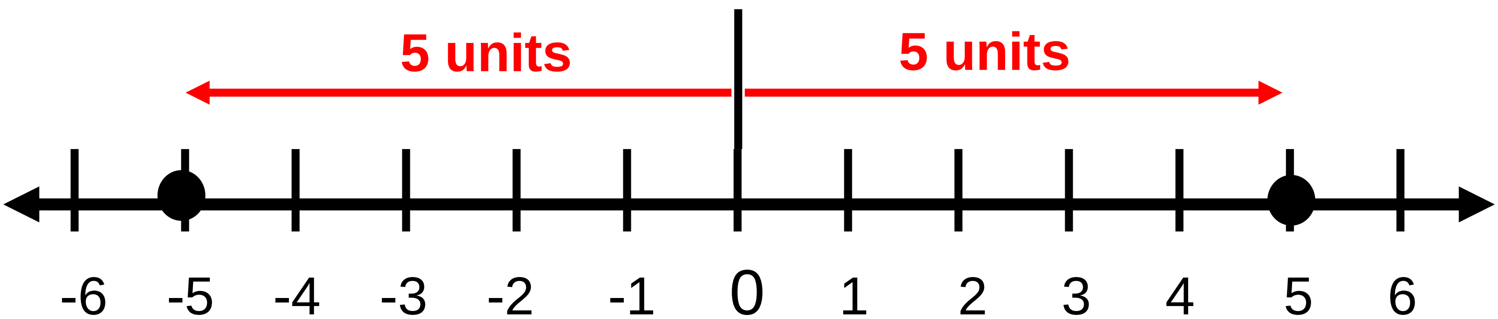
Real Numbers



The set of all rational and irrational numbers

Absolute Value

$|5| = 5$ $|-5| = 5$



The distance between a number and zero

Order of Operations

Grouping Symbols	( ) { } [ ] |absolute value| fraction bar
Exponents	a^n
Multiplication Division	→ **Left to Right**
Addition Subtraction	→ **Left to Right**

Expression

$$x$$

$$-\sqrt{26}$$

$$3^4 + 2m$$

$$3(y + 3.9)^2 - \frac{8}{9}$$

Variable

$$2(y + \sqrt{3})$$

$$9 + x = 2.08$$

$$d = 7c - 5$$

$$A = \pi r^2$$

Coefficient

$$(-4) + 2x$$

$$-7y^2$$

$$\frac{2}{3}ab - \frac{1}{2}$$

$$\pi r^2$$

Term

$$\underbrace{3x} + \underbrace{2y} - \underbrace{8}$$

3 terms

$$\underbrace{-5x^2} - \underbrace{x}$$

2 terms

$$\underbrace{\frac{2}{3}ab}$$

1 term

Scientific Notation

$$a \text{ x } 10^{n}$$

$1 \leq |a| < 10$ and n is an integer

Examples:

Standard Notation	Scientific Notation
17,500,000	$1.75 \text{ x } 10^{7}$
-84,623	$-8.4623 \text{ x } 10^{4}$
0.0000026	$2.6 \text{ x } 10^{-6}$
-0.080029	$-8.0029 \text{ x } 10^{-2}$

Exponential Form

exponent

$$a^{n} = \underbrace{a\cdot a\cdot a\cdot a}_{\text{factors}}\ldots,\ a\neq 0$$

base

Examples:

$$2\cdot 2\cdot 2 = 2^{3} = 8$$

$$n\cdot n\cdot n\cdot n = n^{4}$$

$$3\cdot 3\cdot 3\cdot x\cdot x = 3^{3}x^{2} = 27x^{2}$$

Negative Exponent

$$a^{-n} = \frac{1}{a^n}, \; a \neq 0$$

Examples:

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

$$\frac{x^4}{y^{-2}} = \frac{x^4}{\frac{1}{y^2}} = \frac{x^4}{\frac{1}{y^2}} \cdot \frac{y^2}{y^2} = x^4y^2$$

$$(2-a)^{-2} = \frac{1}{(2-a)^2}, \; a \neq 2$$

Zero Exponent

$$a^0 = 1, \; a \neq 0$$

Examples:

$$(-5)^0 = 1$$

$$(3x + 2)^0 = 1$$

$$(x^2y^{-5}z^8)^0 = 1$$

$$4m^0 = 4 \cdot 1 = 4$$

Product of Powers Property

$$a^m \cdot a^{\mathrm{n}} = a^{m+n}$$

Examples:

$$x^4 \cdot x^2 = x^{4+2} = x^6$$

$$a^3 \cdot a = a^{3+1} = a^4$$

$$w^7 \cdot w^{-4} = w^{7+(-4)} = w^3$$

Power of a Power Property

$$(a^m)^n = a^{m \cdot n}$$

Examples:

$$(y^4)^2 = y^{4 \cdot 2} = y^8$$

$$(g^2)^{-3} = g^{2 \cdot (-3)} = g^{-6} = \frac{1}{g^6}$$

Power of a Product Property

$$(ab)^m = a^m \cdot b^m$$

Examples:

$$(-3ab)^2 = (-3)^2 \cdot a^2 \cdot b^2 = 9a^2b^2$$

$$\frac{-1}{(2x)^3} = \frac{-1}{2^3 \cdot x^3} = \frac{-1}{8x^3}$$

Quotient of Powers Property

$$\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$$

Examples:

$$\frac{x^6}{x^5} = x^{6-5} = x^1 = x$$

$$\frac{y^{-3}}{y^{-5}} = y^{-3-(-5)} = y^2$$

$$\frac{a^4}{a^4} = a^{4-4} = a^0 = 1$$

Power of Quotient Property

$$\left(\frac{a}{b}\right)^{m} = \frac{a^{m}}{b^{m}},\ b \neq 0$$

Examples:

$$\left(\frac{y}{3}\right)^{4} = \frac{y^{4}}{3^{4}}$$

$$\left(\frac{5}{t}\right)^{-3} = \frac{5^{-3}}{t^{-3}} = \frac{\frac{1}{5^{3}}}{\frac{1}{t^{3}}} = \frac{t^{3}}{5^{3}} = \frac{t^{3}}{125}$$

Polynomial

Example	Name	Terms
7 $6x$	monomial	1 term
$3t - 1$ $12xy^3 + 5x^4y$	binomial	2 terms
$2x^2 + 3x - 7$	trinomial	3 terms

Nonexample	Reason
$5m^{n} - 8$	variable exponent
$n^{-3} + 9$	negative exponent

Degree of a Polynomial

The largest exponent or the largest sum of exponents of a term within a polynomial

Example:

$$6a^3 + 3a^2b^3 - 21$$

Term	Degree
$6a^3$	3
$3a^2b^3$	5
-21	0

Degree of polynomial: 5

Leading Coefficient

The coefficient of the first term of a polynomial written in descending order of exponents

Examples:

$$7a^3 - 2a^2 + 8a - 1$$

$$-3n^3 + 7n^2 - 4n + 10$$

$$16t - 1$$

Add Polynomials

Combine like terms.

Example:

$$(2g^2 + 6g - 4) + (g^2 - g)$$

$$= 2g^2 + 6g - 4 + g^2 - g$$

(Group like terms and add.)

$$= (2g^2 + g^2) + (6g - g) - 4$$

$$= 3g^2 + 5g^2 - 4$$

Add Polynomials

Combine <u>like</u> terms.

Example:

$$(2g^3 + 6g^2 - 4) + (g^3 - g - 3)$$

(Align like terms and add.)

$$\begin{array}{rrrr} 2g^3 & + 6g^2 & & - 4 \\ + \quad g^3 & & - g & - 3 \\ \hline 3g^3 & + 6g^2 & - g & - 7 \end{array}$$

Subtract Polynomials

Add the inverse.

Example:

$$(4x^2 + 5) - (-2x^2 + 4x - 7)$$

(Add the inverse.)

$$= (4x^2 + 5) + (2x^2 - 4x + 7)$$

$$= 4x^2 + 5 + 2x^2 - 4x + 7$$

(Group like terms and add.)

$$= (4x^2 + 2x^2) - 4x + (5 + 7)$$

$$= 6x^2 - 4x + 12$$

Subtract Polynomials

Add the inverse.

Example:

$$(4x^2 + 5) - (-2x^2 + 4x - 7)$$

(Align like terms then add the inverse and add the like terms.)

$$\begin{array}{rrr} 4x^2 & & +5 \\ \underline{-(2x^2} & \underline{+4x} & \underline{-7)} \end{array} \longrightarrow \begin{array}{rrr} 4x^2 & & +5 \\ \underline{+\ 2x^2} & \underline{-4x} & \underline{+7} \\ 6x^2 & -4x & +12 \end{array}$$

Multiply Polynomials

Apply the distributive property.

$$(a + b)(d + e + f)$$

$$(a + b)(d + e + f)$$

$$= a(d + e + f) + b(d + e + f)$$

$$= ad + ae + af + bd + be + bf$$

Multiply Binomials

Apply the distributive property.

$$(a + b)(c + d) =$$
$$a(c + d) + b(c + d) =$$
$$ac + ad + bc + bd$$

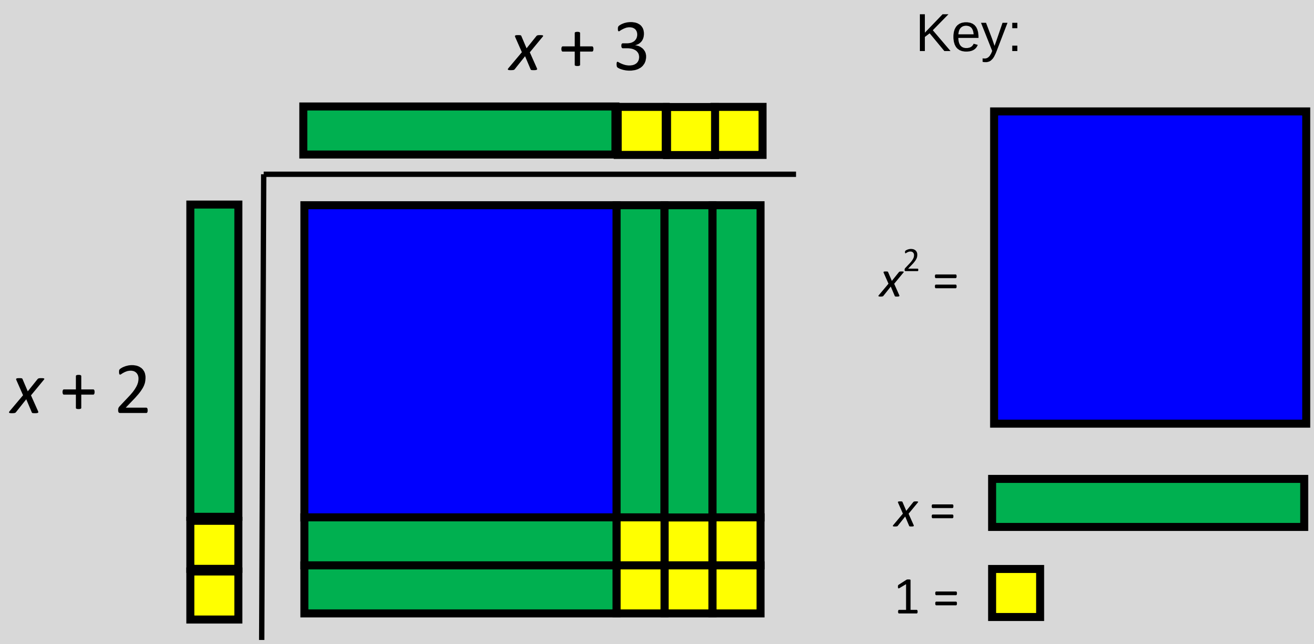
Example: $(x + 3)(x + 2)$

$$= x(x + 2) + 3(x + 2)$$
$$= x^2 + 2x + 3x + 6$$
$$= x^2 + 5x + 6$$

Multiply Binomials

Apply the distributive property.

Example: $(x + 3)(x + 2)$



$$x^2 + 2x + 3x + 6 = x^2 + 5x + 6$$

Multiply Binomials

Apply the distributive property.

Example: $(x + 8)(2x - 3)$

$= (x + 8)(2x + -3)$

	$2x$	$+$	-3
x	$2x^2$		$-3x$
$+$			
8	$8x$		-24

$$2x^2 + 8x + -3x + -24 = 2x^2 + 5x - 24$$

Multiply Binomials: Squaring a Binomial

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Examples:

$$(3m + n)^2 = 9m^2 + 2(3m)(n) + n^2$$
$$= 9m^2 + 6mn + n^2$$

$$(y - 5)^2 = y^2 - 2(5)(y) + 25$$
$$= y^2 - 10y + 25$$

Multiply Binomials: Sum and Difference

$$(a + b)(a - b) = a^2 - b^2$$

Examples:

$$(2b + 5)(2b - 5) = 4b^2 - 25$$

$$\begin{aligned}(7 - w)(7 + w) &= 49 + 7w - 7w - w^2 \\ &= 49 - w^2\end{aligned}$$

Factors of a Monomial

The number(s) and/or variable(s) that are multiplied together to form a monomial

Examples:	Factors	Expanded Form
$5b^2$	$5 \cdot b^2$	$5 \cdot b \cdot b$
$6x^2y$	$6 \cdot x^2 \cdot y$	$2 \cdot 3 \cdot x \cdot x \cdot y$
$\frac{-5p^2q^3}{2}$	$\frac{-5}{2} \cdot p^2 \cdot q^3$	$\frac{1}{2} \cdot (-5) \cdot p \cdot p \cdot q \cdot q \cdot q$

Factoring: Greatest Common Factor

Find the greatest common factor (GCF) of all terms of the polynomial and then apply the distributive property.

Example: $20a^4 + 8a$

$$2 \cdot 2 \cdot 5 \cdot a \cdot a \cdot a \cdot a + 2 \cdot 2 \cdot 2 \cdot a$$

$$\text{GCF} = \overbrace{2 \cdot 2 \cdot a}^{\text{common factors}} = 4a$$

$$20a^4 + 8a = 4a(5a^3 + 2)$$

Factoring: Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

Examples:

$$x^2 + 6x + 9 = x^2 + 2 \cdot 3 \cdot x + 3^2$$
$$= (x + 3)^2$$

$$4x^2 - 20x + 25 = (2x)^2 - 2 \cdot 2x \cdot 5 + 5^2$$
$$= (2x - 5)^2$$

Factoring: Difference of Two Squares

$$a^2 - b^2 = (a + \mathrm{b})(a - b)$$

Examples:

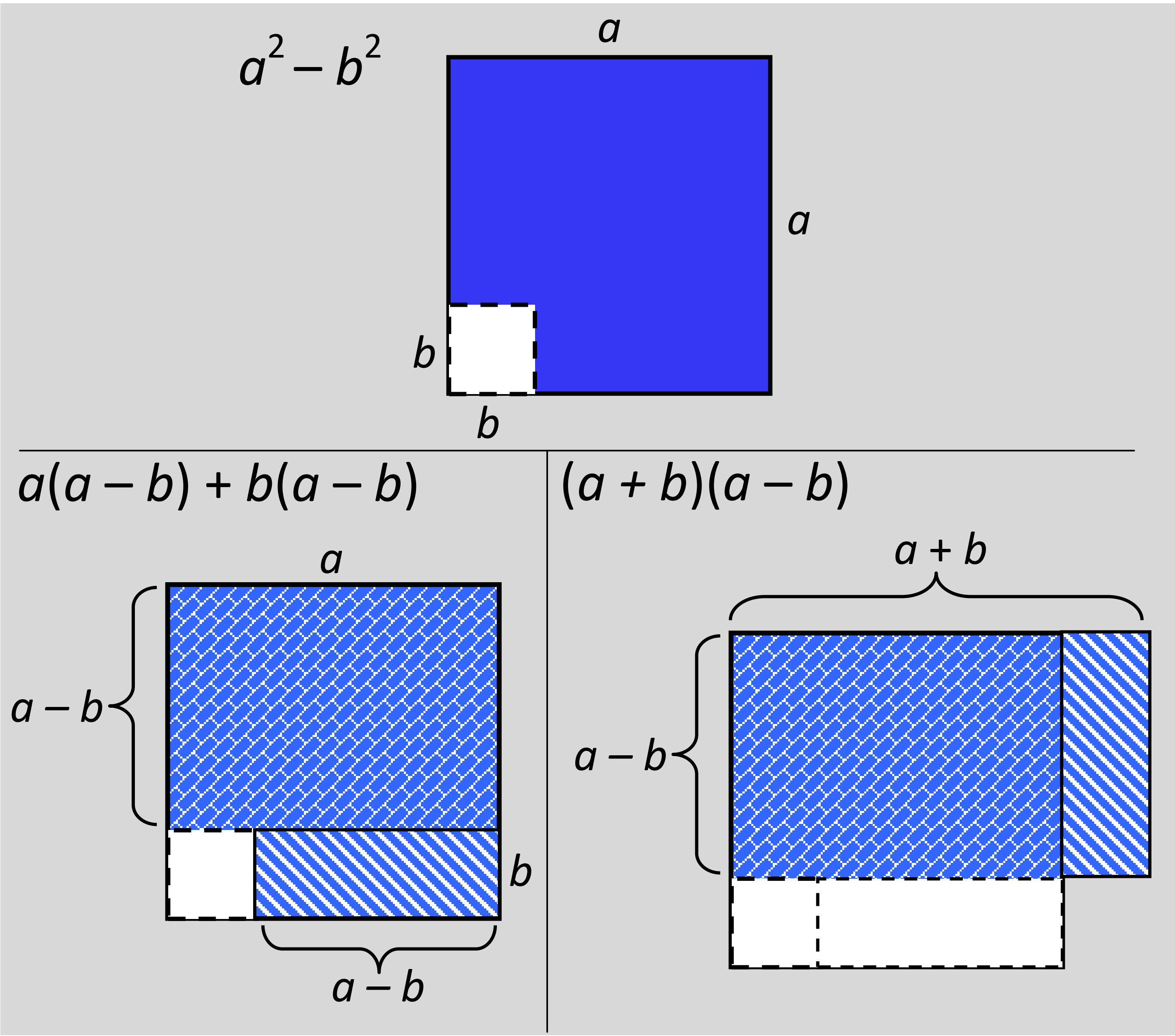
$$x^2 - 49 = x^2 - 7^2 = (x + 7)(x - 7)$$

$$4 - n^2 = 2^2 - n^2 = (2 - n)\ (2 + n)$$

$$9x^2 - 25y^2 = (3x)^2 - (5y)^2$$
$$= (3x + 5y)(3x - 5y)$$

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$



Divide Polynomials

Divide each term of the dividend by the monomial divisor

Example:

$$(12x^3 - 36x^2 + 16x) \div 4x$$

$$= \frac{12x^3 - 36x^2 + 16x}{4x}$$

$$= \frac{12x^3}{4x} - \frac{36x^2}{4x} + \frac{16x}{4x}$$

$$= 3x^2 - 9x + 4$$

Divide Polynomials by Binomials

Factor and simplify

Example:

$$(7w^2 + 3w - 4) \div (w + 1)$$

$$= \frac{7w^2 + 3w - 4}{w + 1}$$

$$= \frac{\left(7w - 4\right)(w + 1)}{w + 1}$$

$$= 7w - 4$$

Prime Polynomial

Cannot be factored into a product of lesser degree polynomial factors

Example
r
$3t + 9$
$x^2 + 1$
$5y^2 - 4y + 3$

Nonexample	Factors
$x^2 - 4$	$(x + 2)(x - 2)$
$3x^2 - 3x + 6$	$3(x + 1)(x - 2)$
x^3	$x \cdot x^2$

Square Root

$$\sqrt{x^2}$$

radical symbol

radicand or argument

Simply square root expressions.

Examples:

$$\sqrt{9x^2} = \sqrt{3^2 \cdot x^2} = \sqrt{(3x)^2} = 3x$$

$$-\sqrt{(x-3)^2} = -(x-3) = -x+3$$

Squaring a number and taking a square root are inverse operations.

Cube Root

index

$$\sqrt[3]{x^3}$$

radical symbol

radicand or argument

Simplify cube root expressions.

Examples:

$$\sqrt[3]{64} = \sqrt[3]{4^3} = 4$$

$$\sqrt[3]{-27} = \sqrt[3]{(-3)^3} = -3$$

$$\sqrt[3]{x^3} = x$$

Cubing a number and taking a cube root are inverse operations.

Product Property of Radicals

The square root of a product equals the product of the square roots of the factors.

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

$a \geq 0$ and $b \geq 0$

Examples:

$$\sqrt{4x} = \sqrt{4} \cdot \sqrt{x} = 2\sqrt{x}$$

$$\sqrt{5a^3} = \sqrt{5} \cdot \sqrt{a^3} = a\sqrt{5a}$$

$$\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$$

Quotient Property of Radicals

The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$a \geq 0$ and $b > 0$

Example:

$$\sqrt{\frac{5}{y^2}} = \frac{\sqrt{5}}{\sqrt{y^2}} = \frac{\sqrt{5}}{y},\ y \neq 0$$

Zero Product Property

If $ab = 0$,
then $a = 0$ or $b = 0$.

Example:

$$(x + 3)(x - 4) = 0$$

$$(x + 3) = 0 \text{ or } (x - 4) = 0$$

$$x = -3 \text{ or } x = 4$$

The solutions are -3 and 4, also called roots of the equation.

Solutions or Roots

$$x^2 + 2x = 3$$

Solve using the zero product property.

$$x^2 + 2x - 3 = 0$$

$$(x + 3)(x - 1) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = -3 \text{ or } x = 1$$

The solutions or roots of the polynomial equation are -3 and 1.

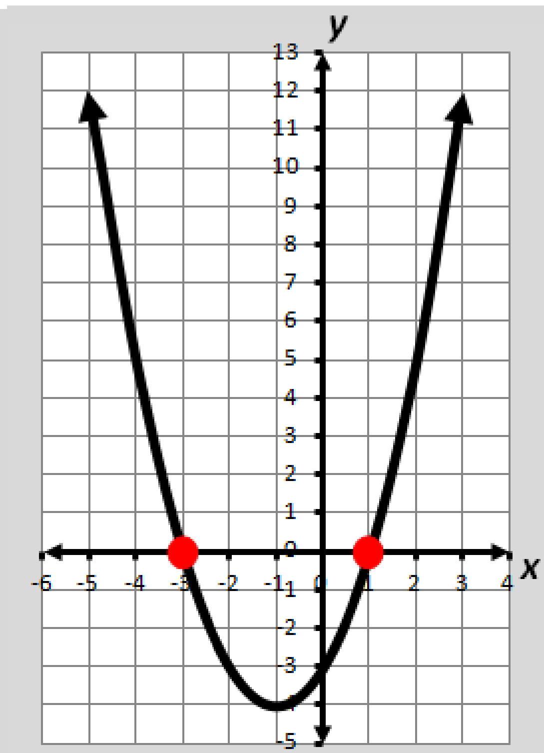
Zeros

The zeros of a function f(x) are the values of x where the function is equal to zero.

$f(x) = x^2 + 2x - 3$
Find $f(x) = 0$.

$0 = x^2 + 2x - 3$
$0 = (x + 3)(x - 1)$
$x = -3$ or $x = 1$

The zeros are -3 and 1 located at (-3,0) and (1,0).



The zeros of a function are also the solutions or roots of the related equation.

x-Intercepts

The *x*-intercepts of a graph are located where the graph crosses the x-axis and where $f(x) = 0$.

$$f(x) = x^2 + 2x - 3$$

$$0 = (x + 3)(x - 1)$$

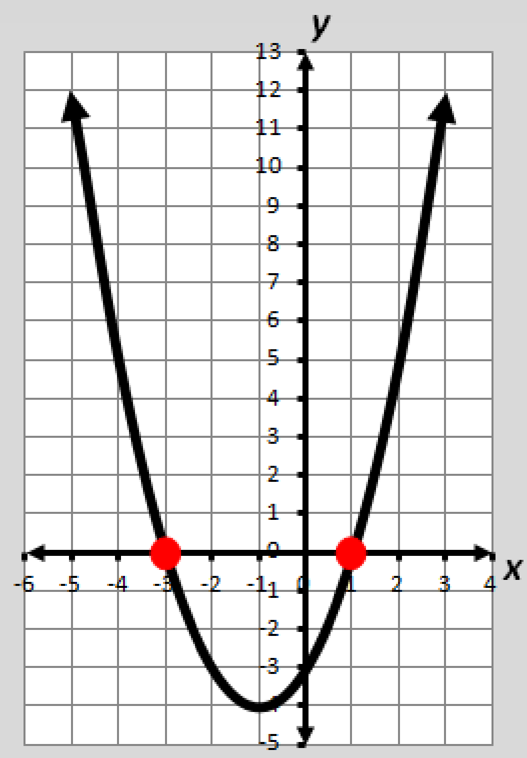
$$0 = x + 3 \text{ or } 0 = x - 1$$

$$x = -3 \text{ or } x = 1$$

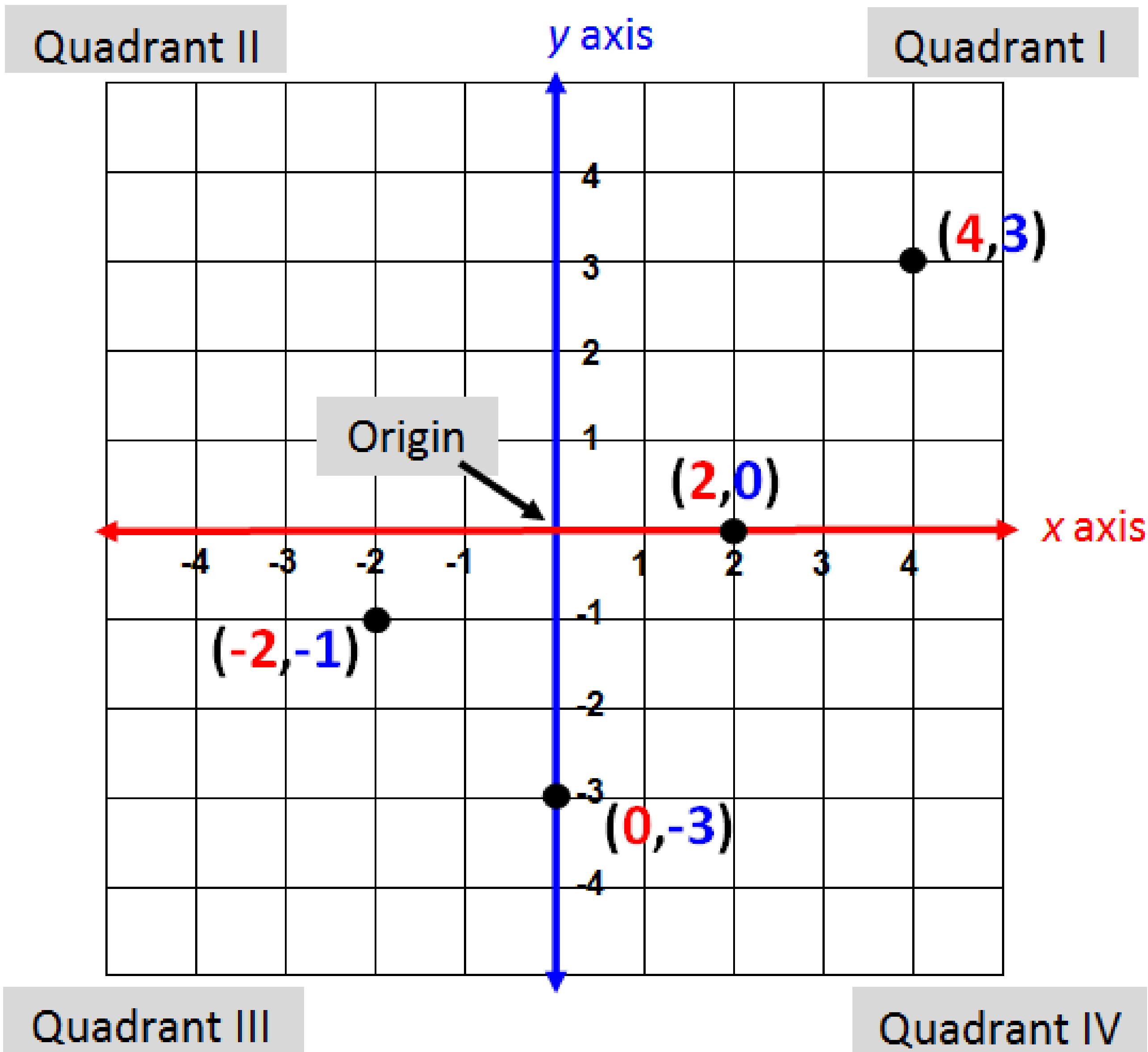
The zeros are -3 and 1.

The *x*-intercepts are:

- -3 or (-3,0)
- 1 or (1,0)



Coordinate Plane



ordered pair (x,y)

(abscissa, ordinate)

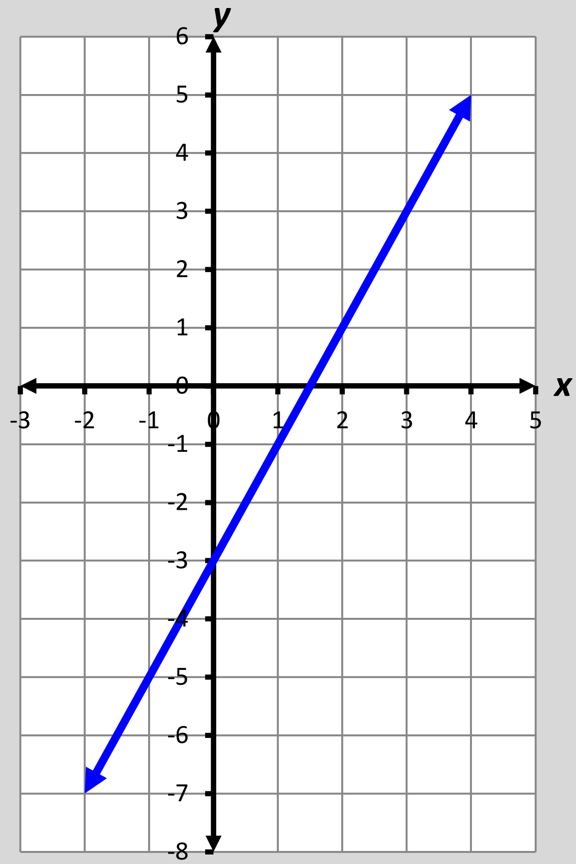
Linear Equation

$Ax + By = C$

(A, B and C are integers; A and B cannot both equal zero.)

Example:

$-2x + y = -3$



The graph of the linear equation is a straight line and represents all solutions (x, y) of the equation.

Linear Equation: Standard Form

$$Ax + By = C$$

(A, B, and C are integers;
A and B cannot both equal zero.)

Examples:

$$4x + 5y = -24$$

$$x - 6y = 9$$

Literal Equation

A formula or equation which consists primarily of variables

Examples:

$$ax + \mathrm{b} = \mathrm{c}$$

$$A = \frac{1}{2}bh$$

$$V = lwh$$

$$F = \frac{9}{5}C + 32$$

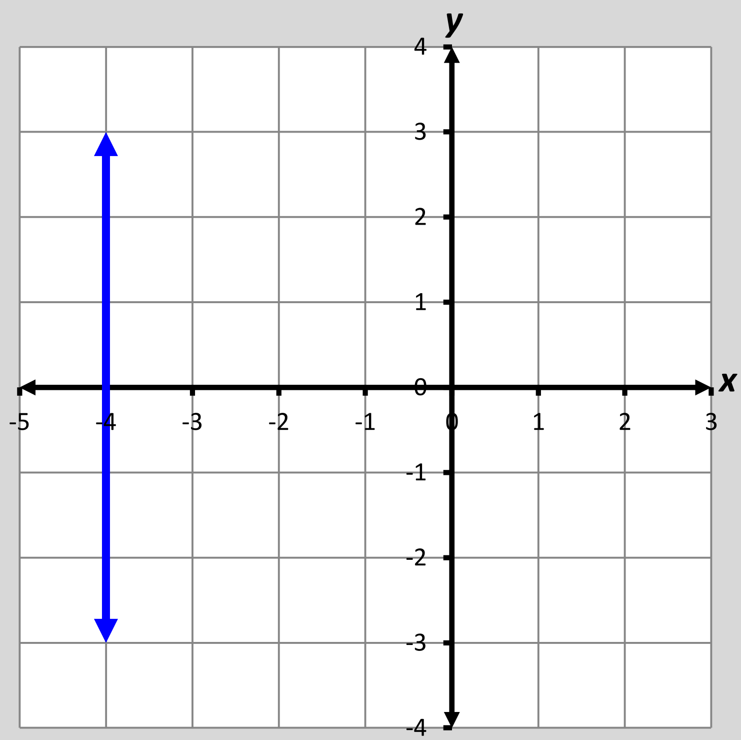
$$A = \pi r^2$$

Vertical Line

$$x = a$$

(where a can be any real number)

Example: $x = -4$



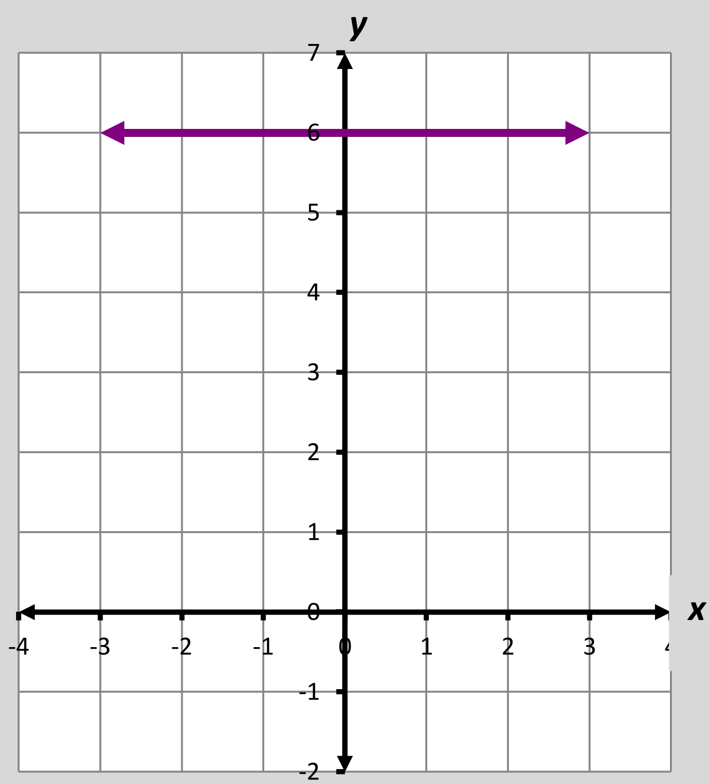
Vertical lines have an undefined slope.

Horizontal Line

$$y = c$$

(where c can be any real number)

Example: $y = 6$



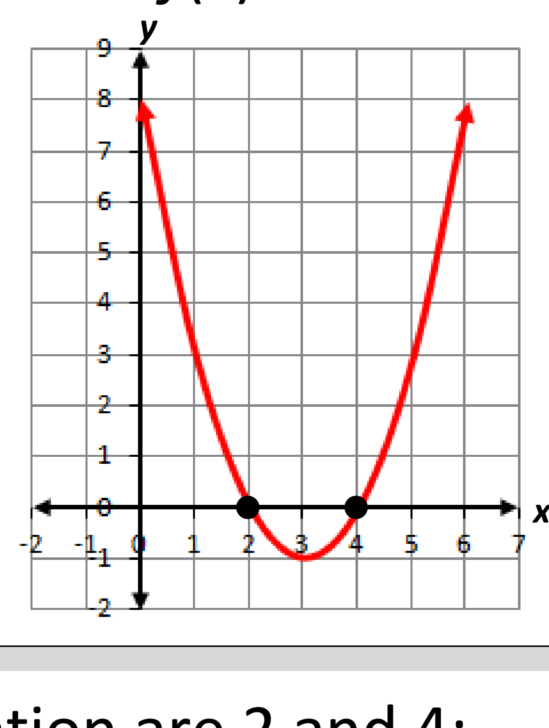
Horizontal lines have a slope of 0.

Quadratic Equation

$$ax^2 + bx + c = 0$$

$$a \neq 0$$

Example: $x^2 - 6x + 8 = 0$

Solve by factoring	Solve by graphing
$x^2 - 6x + 8 = 0$ $(x - 2)(x - 4) = 0$ $(x - 2) = 0$ or $(x - 4) = 0$ $x = 2$ or $x = 4$	Graph the related function $f(x) = x^2 - 6x + 8$. 

Solutions to the equation are 2 and 4; the *x*-coordinates where the curve crosses the x-axis.

Quadratic Equation

$$ax^2 + bx + c = 0$$

$a \neq 0$

Example solved by factoring:

$x^2 - 6x + 8 = 0$	Quadratic equation
$(x - 2)(x - 4) = 0$	Factor
$(x - 2) = 0$ or $(x - 4) = 0$	Set factors equal to 0
$x = 2$ or $x = 4$	Solve for x

Solutions to the equation are 2 and 4.

Quadratic Equation

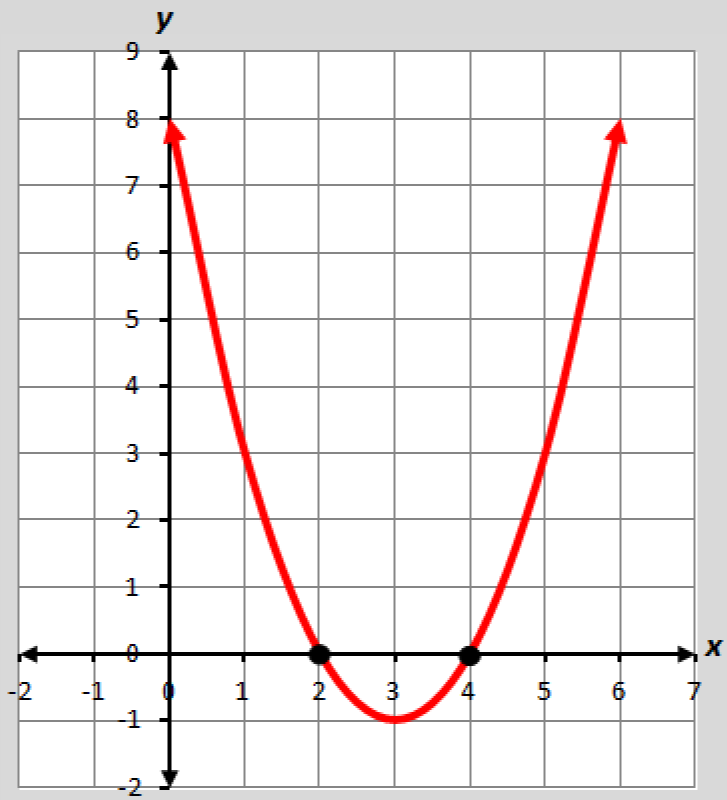
$$ax^2 + bx + c = 0$$

$$a \neq 0$$

Example solved by graphing:

$$x^2 - 6x + 8 = 0$$

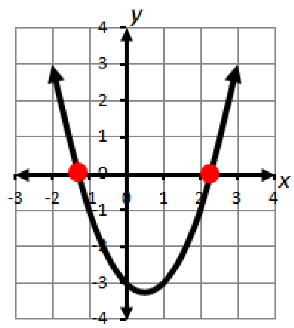
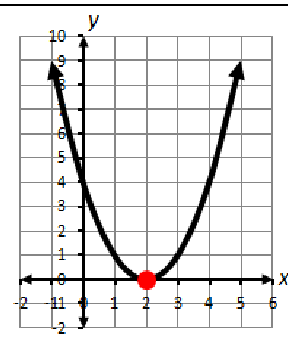
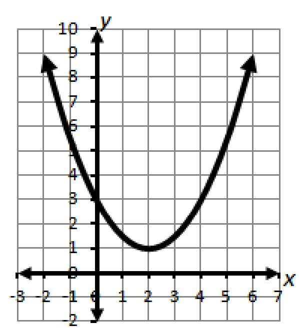
Graph the related function $f(x) = x^2 - 6x + 8$.



Solutions to the equation are the x-coordinates (2 and 4) of the points where the curve crosses the x-axis.

Quadratic Equation: Number of Real Solutions

$$ax^2 + bx + c = 0, \; a \neq 0$$

Examples	Graphs	Number of Real Solutions/Roots
$x^2 - x = 3$		2
$x^2 + 16 = 8x$		1 distinct root with a multiplicity of two
$2x^2 - 2x + 3 = 0$		0

Identity Property of Addition

$$a + 0 = 0 + a = a$$

Examples:

$$3.8 + 0 = 3.8$$

$$6x + 0 = 6x$$

$$0 + (-7 + r) = -7 + r$$

Zero is the additive identity.

Inverse Property of Addition

$$a + (-a) = (-a) + a = 0$$

Examples:

$$4 + (-4) = 0$$

$$0 = (-9.5) + 9.5$$

$$x + (-x) = 0$$

$$0 = 3y + (-3y)$$

Commutative Property of Addition

$$a + b = b + a$$

Examples:

$$2.76 + 3 = 3 + 2.76$$

$$x + 5 = 5 + \mathrm{x}$$

$$(a + 5) - 7 = (5 + a) - 7$$

$$11 + (\mathrm{b} - 4) = (\mathrm{b} - 4) + 11$$

Associative Property of Addition

$$(a + b) + c = a + (b + c)$$

Examples:

$$\left(5 + \frac{3}{5}\right) + \frac{1}{10} = 5 + \left(\frac{3}{5} + \frac{1}{10}\right)$$

$$3x + (2x + 6y) = (3x + 2x) + 6y$$

Identity Property of Multiplication

$$a \cdot 1 = 1 \cdot a = a$$

Examples:

$$3.8\,(1) = 3.8$$

$$6x \cdot 1 = 6x$$

$$1(-7) = -7$$

One is the multiplicative identity.

Inverse Property of Multiplication

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$$

$$a \neq 0$$

Examples:

$$7 \cdot \frac{1}{7} = 1$$

$$\frac{5}{x} \cdot \frac{x}{5} = 1, \; x \neq 0$$

$$\frac{-1}{3} \cdot (-3p) = 1p = p$$

The multiplicative inverse of a is $\frac{1}{a}$.

Commutative Property of Multiplication

$$ab = ba$$

Examples:

$$(-8)\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)(-8)$$

$$y \cdot 9 = 9 \cdot y$$

$$4(2x \cdot 3) = 4(3 \cdot 2x)$$

$$8 + 5x = 8 + x \cdot 5$$

Associative Property of Multiplication

$$(ab)c = a(bc)$$

Examples:

$$(1 \cdot 8) \cdot 3\frac{3}{4} = 1 \cdot (8 \cdot 3\frac{3}{4})$$

$$(3x)x = 3(x \cdot x)$$

Distributive Property

$$a(b + c) = ab + ac$$

Examples:

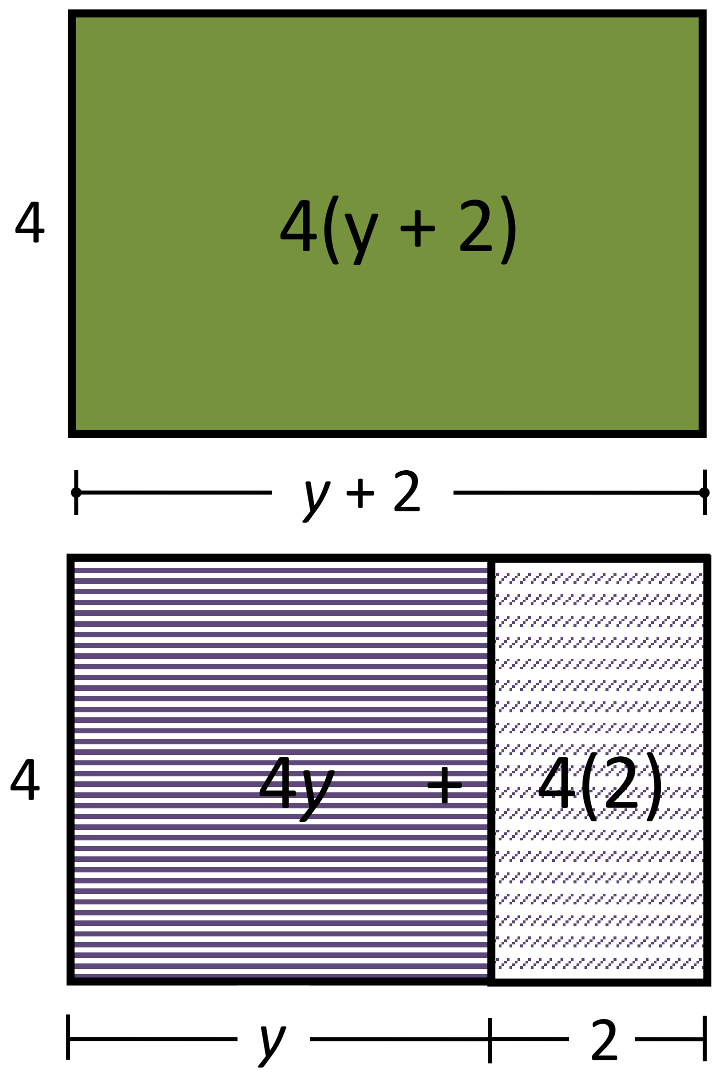
$$5\left(y - \frac{1}{3}\right) = (5 \cdot y) - \left(5 \cdot \frac{1}{3}\right)$$

$$2 \cdot x + 2 \cdot 5 = 2(x + 5)$$

$$3.1a + (1)(a) = (3.1 + 1)a$$

Distributive Property

$$4(y + 2) = 4y + 4(2)$$



Multiplicative Property of Zero

$$a \cdot 0 = 0 \text{ or } 0 \cdot a = 0$$

Examples:

$$8\frac{2}{3} \cdot 0 = 0$$

$$0 \cdot (-13y - 4) = 0$$

Substitution Property

If $a = b$, then b can replace a in a given equation or inequality.

Examples:

Given	Given	Substitution
$r = 9$	$3r = 27$	$3(9) = 27$
$b = 5a$	$24 < b + 8$	$24 < 5a + 8$
$y = 2x + 1$	$2y = 3x - 2$	$2(2x + 1) = 3x - 2$

Reflexive Property of Equality

$$a = a$$

a is any real number

Examples:

$$-4 = -4$$

$$3.4 = 3.4$$

$$9y = 9y$$

Symmetric Property of Equality

If $a = b$, then $b = a$.

Examples:

If $12 = r$, then $r = 12$.

If $-14 = z + 9$, then $z + 9 = -14$.

If $2.7 + y = x$, then $x = 2.7 + y$.

Transitive Property of Equality

If $a = b$ and $b = c$,
then $a = c$.

Examples:

If $4x = 2y$ and $2y = 16$,
then $4x = 16$.

If $x = y - 1$ and $y - 1 = -3$,
then $x = -3$.

Inequality

An algebraic sentence comparing two quantities

Symbol	Meaning
$<$	less than
$\leq$	less than or equal to
$>$	greater than
$\geq$	greater than or equal to
$\neq$	not equal to

Examples:

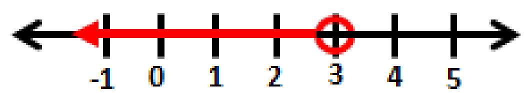
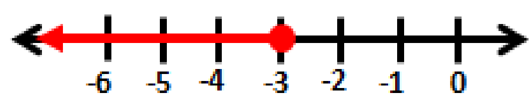
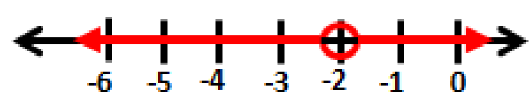
$$-10.5 > -9.9 - 1.2$$

$$8 > 3t + 2$$

$$x - 5y \geq -12$$

$$r \neq 3$$

Graph of an Inequality

Symbol	Examples	Graph
$<$ or $>$	$x < 3$	
$\leq$ or $\geq$	$-3 \geq y$	
$\neq$	$t \neq -2$	

Transitive Property of Inequality

If	Then
$a < b$ and $b < c$	$a < c$
$a > b$ and $b > c$	$a > c$

Examples:

If $4x < 2y$ and $2y < 16$,
then $4x < 16$.

If $x > y - 1$ and $y - 1 > 3$,
then $x > 3$.

Addition/Subtraction Property of Inequality

If	Then
$a > b$	$a + c > b + c$
$a \geq b$	$a + c \geq b + c$
$a < b$	$a + c < b + c$
$a \leq b$	$a + c \leq b + c$

Example:

$$d - 1.9 \geq -8.7$$

$$d - 1.9 + 1.9 \geq -8.7 + 1.9$$

$$d \geq -6.8$$

Multiplication Property of Inequality

If	Case	Then
$a < b$	$c > 0$, positive	$ac < bc$
$a > b$	$c > 0$, positive	$ac > bc$
$a < b$	$c < 0$, negative	$ac > bc$
$a > b$	$c < 0$, negative	$ac < bc$

Example: if $c = -2$

$$5 > -3$$

$$5(-2) < -3(-2)$$

$$-10 < 6$$

Division Property of Inequality

If	Case	Then
$a < b$	$c > 0$, positive	$\frac{a}{c} < \frac{b}{c}$
$a > b$	$c > 0$, positive	$\frac{a}{c} > \frac{b}{c}$
$a < b$	$c < 0$, negative	$\frac{a}{c} > \frac{b}{c}$
$a > b$	$c < 0$, negative	$\frac{a}{c} < \frac{b}{c}$

Example: if $c = -4$

$$-90 \geq -4t$$

$$\frac{-90}{-4} \leq \frac{-4t}{-4}$$

$$22.5 \leq t$$

Linear Equation: Slope-Intercept Form

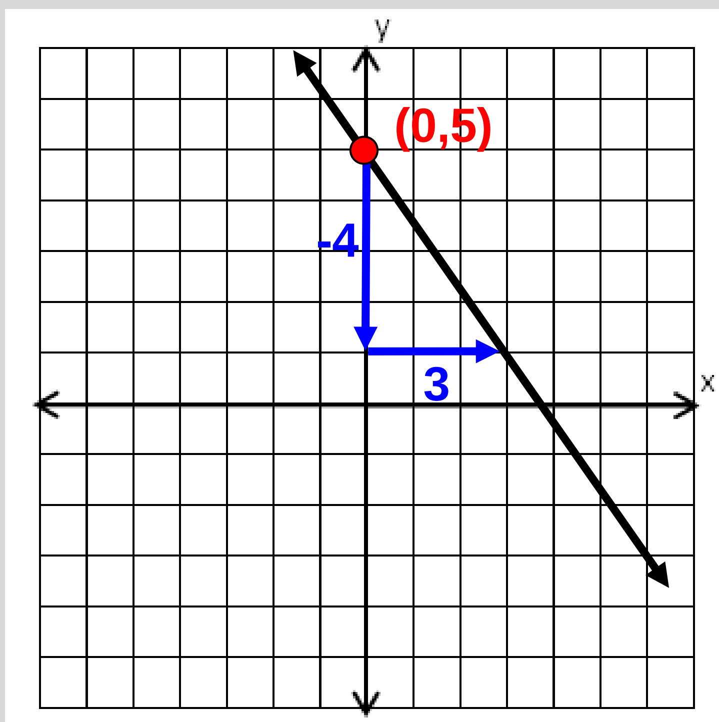
$$y = mx + b$$

(slope is m and y-intercept is b)

Example: $y = \frac{-4}{3}x + 5$

$m = \frac{-4}{3}$

$b = -5$



Linear Equation: Point-Slope Form

$$y - y_1 = m(x - x_1)$$

where m is the slope and (x_1, y_1) is the point

Example:

Write an equation for the line that passes through the point $(^-4,1)$ and has a slope of 2.

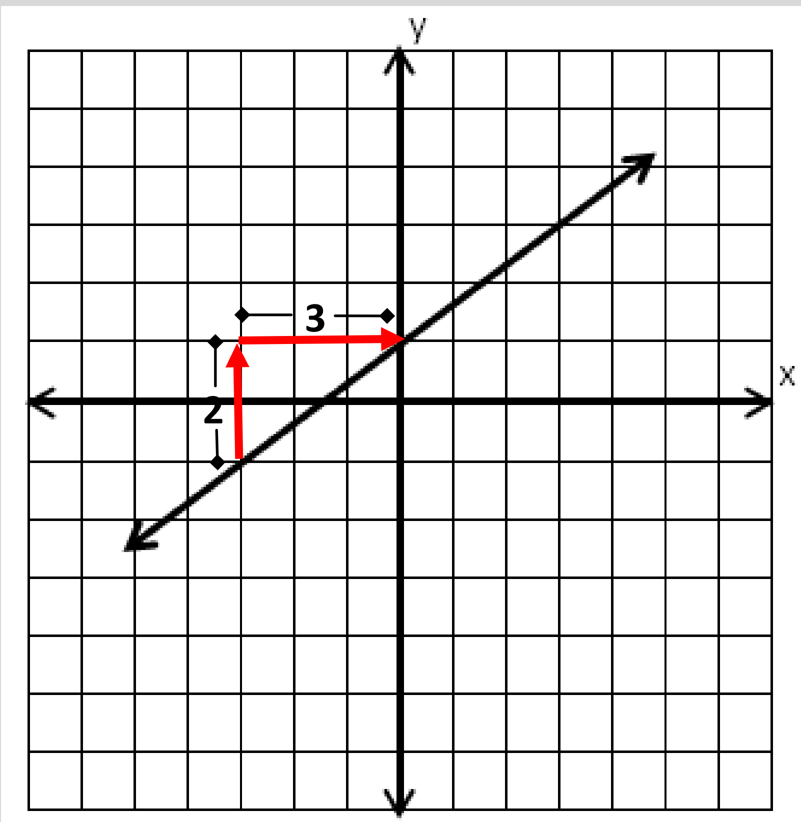
$$y - 1 = 2(x - {}^-4)$$

$$y - 1 = 2(x + 4)$$

$$y = 2x + 9$$

Slope

A number that represents the rate of change in y for a unit change in x

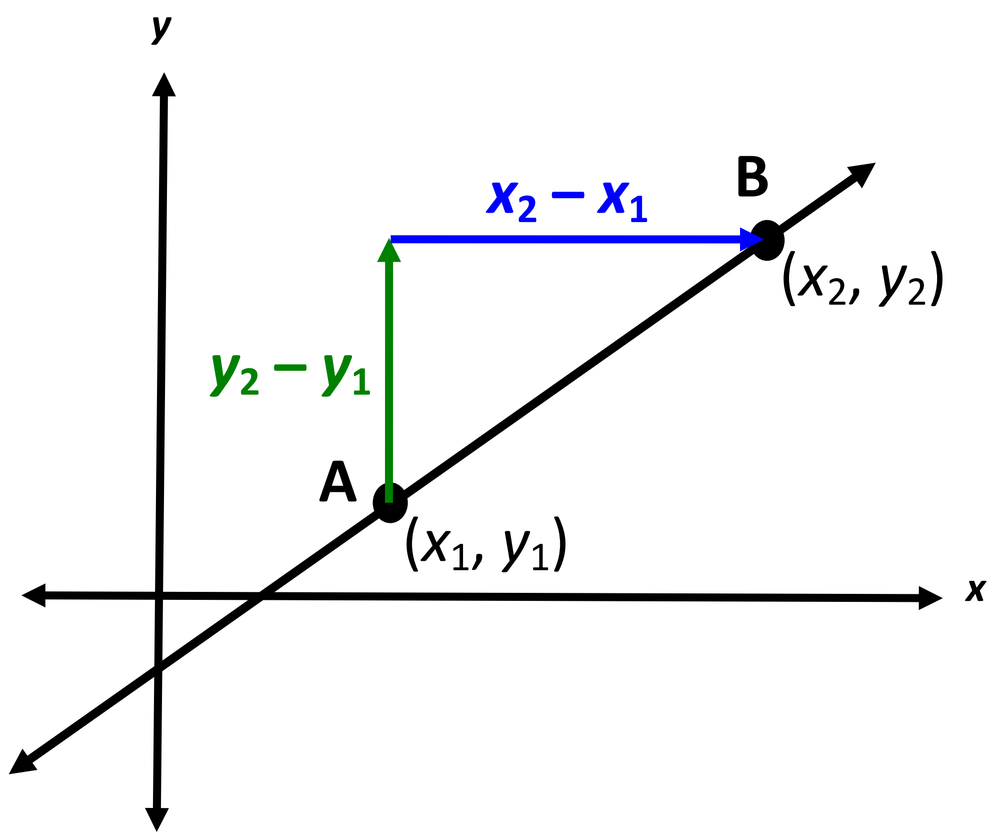


Slope = $\frac{2}{3}$

The slope indicates the steepness of a line.

Slope Formula

The ratio of vertical change to horizontal change

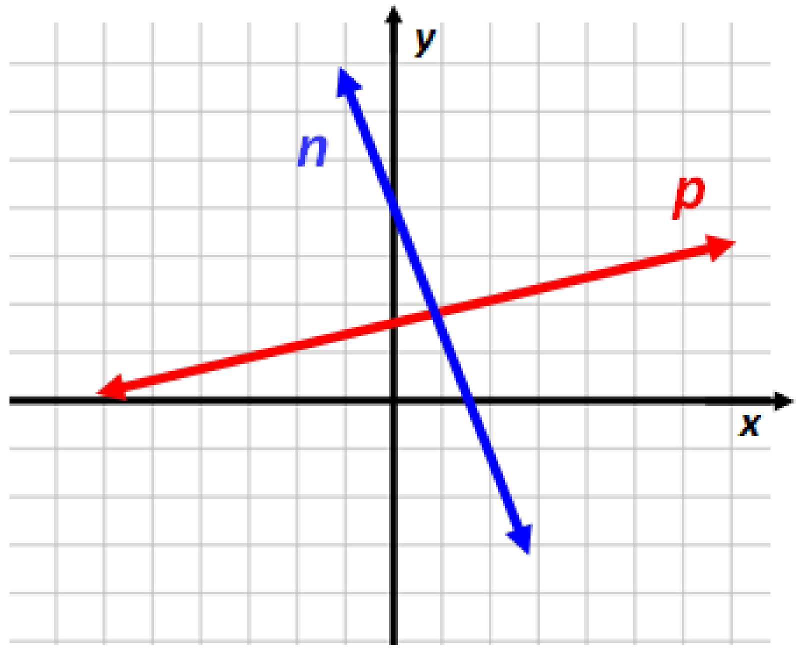


$$\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slopes of Lines

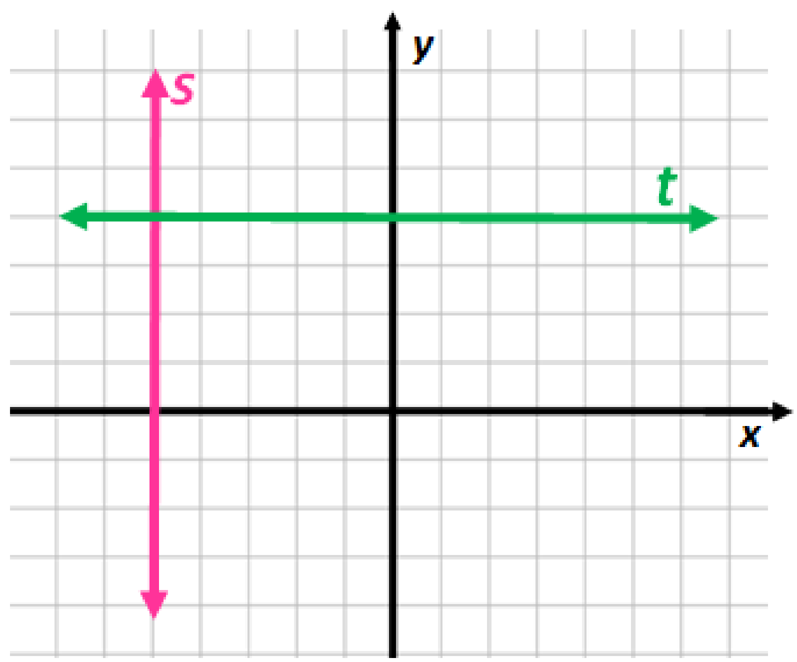
Line ***p***
has a positive
slope.

Line ***n***
has a negative
slope.



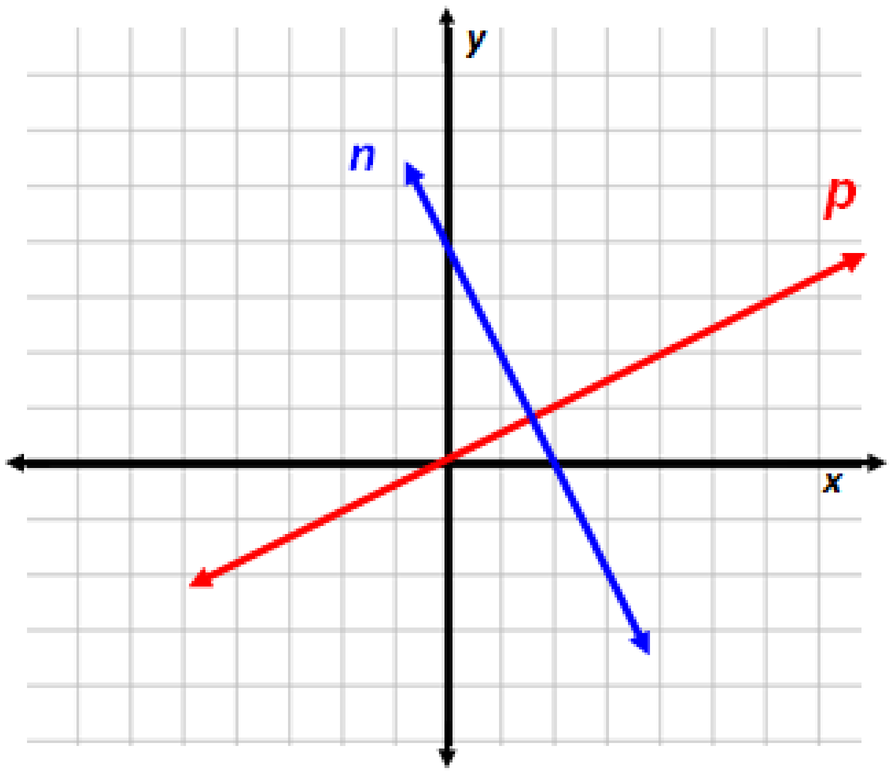
Vertical line **s** has
an undefined
slope.

Horizontal line ***t***
has a zero slope.



Perpendicular Lines

Lines that intersect to form a right angle



Perpendicular lines (not parallel to either of the axes) have slopes whose product is -1.

Example:

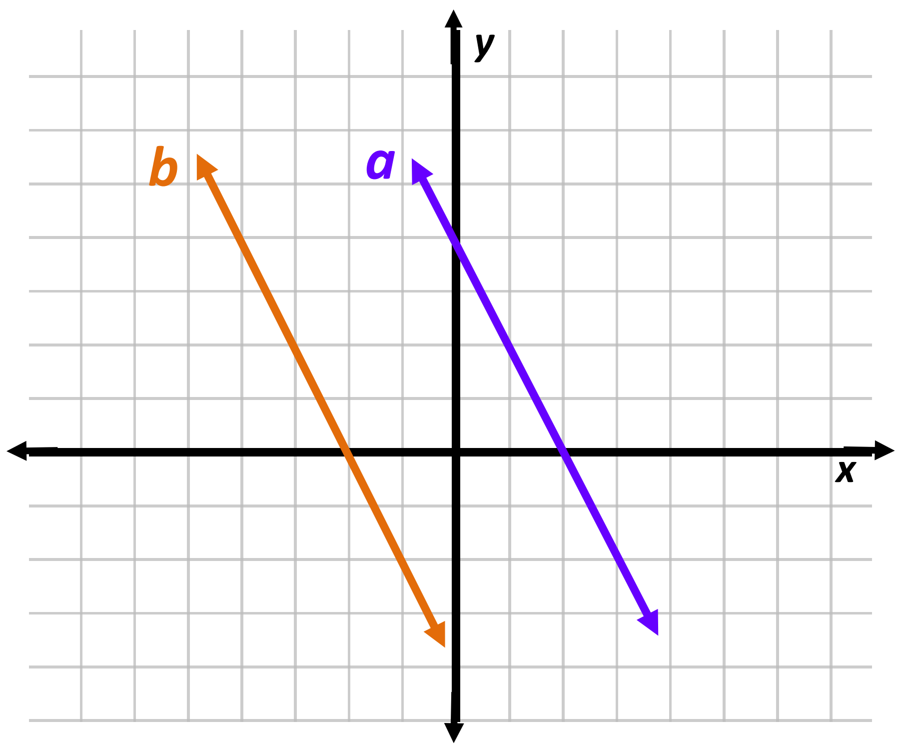
The slope of line *n* = -2. The slope of line *p* = $\frac{1}{2}$.

$-2 \cdot \frac{1}{2} = -1$, therefore, *n* is perpendicular to *p*.

Parallel Lines

Lines in the same plane that do not intersect are parallel.

Parallel lines have the same slopes.



Example:

The slope of line *a* = -2.

The slope of line *b* = -2.

-2 = -2, therefore, *a* is parallel to *b*.

Mathematical Notation

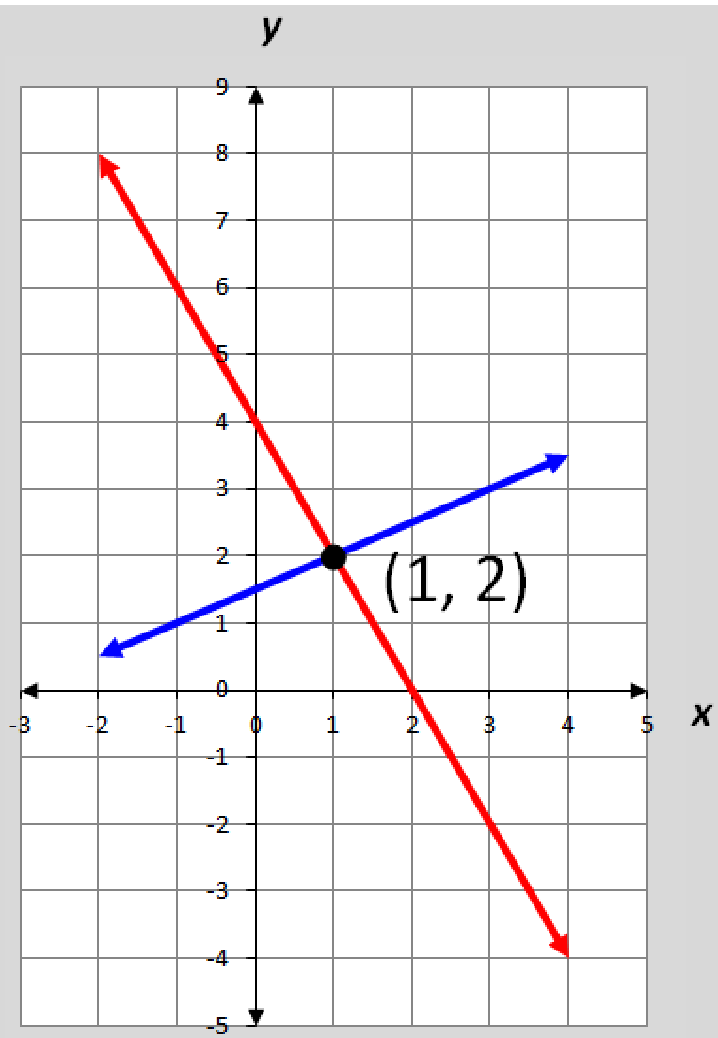
Set Builder Notation	Read	Other Notation
$\{x \mid 0 < x \leq 3\}$	The set of all *x* such that *x* is greater than or equal to 0 and *x* is less than 3.	$0 < x \leq 3$ $(0, 3]$
$\{y: y \geq -5\}$	The set of all *y* such that *y* is greater than or equal to -5.	$y \geq -5$ $[-5, \infty)$

System of Linear Equations

Solve by graphing:

$$\begin{cases} -x + 2y = 3 \\ 2x + y = 4 \end{cases}$$

The solution, (1, 2), is the only ordered pair that satisfies both equations (the point of intersection).



System of Linear Equations

Solve by substitution:

$$\begin{cases} x + 4y = 17 \\ y = x - 2 \end{cases}$$

Substitute x – 2 for y in the first equation.

$$x + 4(x - 2) = 17$$

$$x = 5$$

Now substitute 5 for x in the second equation.

$$y = 5 - 2$$

$$y = 3$$

The solution to the linear system is (5, 3),

the ordered pair that satisfies both equations.

System of Linear Equations

Solve by elimination:

$$\begin{cases} -5x - 6y = 8 \\ 5x + 2y = 4 \end{cases}$$

Add or subtract the equations to eliminate one variable.

$$\begin{array}{r} -5x - 6y = 8 \\ +\ 5x + 2y = 4 \\ \hline -4y = 12 \\ y = -3 \end{array}$$

Now substitute -3 for y in either original equation to find the value of x, the eliminated variable.

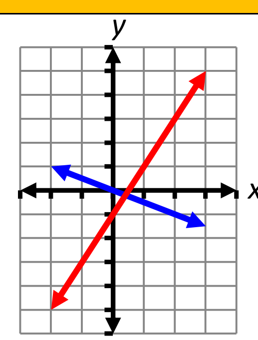
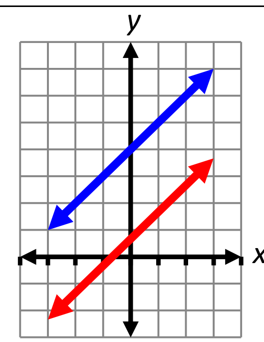
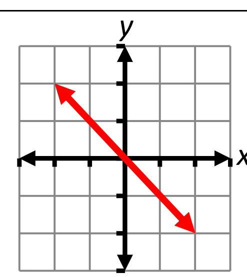
$$-5x - 6(-3) = 8$$

$$x = 2$$

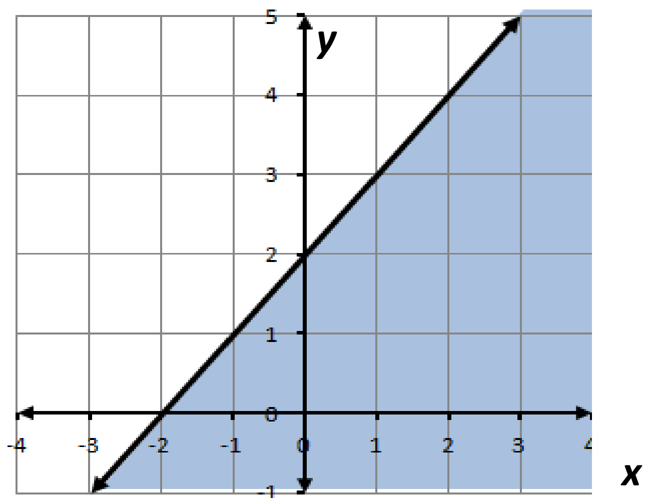
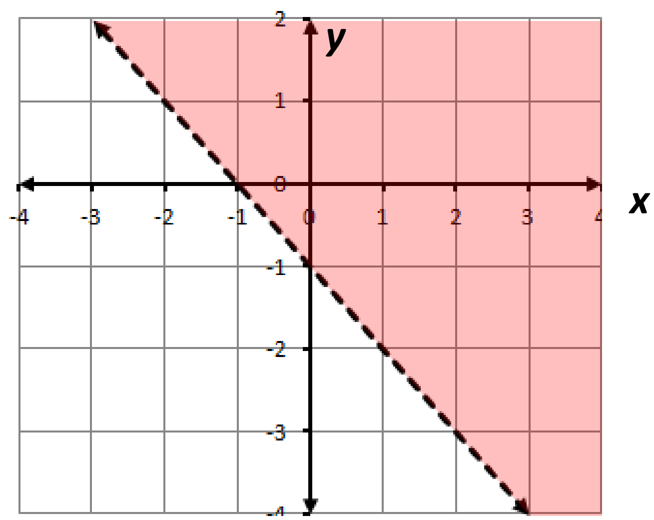
The solution to the linear system is (2,-3), the ordered pair that satisfies both equations.

System of Linear Equations

Identifying the Number of Solutions

Number of Solutions	Slopes and y-intercepts	Graph
One solution	Different slopes	
No solution	Same slope and different y-intercepts	
Infinitely many solutions	Same slope and same y-intercepts	

Graphing Linear Inequalities

Example	Graph
$y \leq x + 2$	
$y > -x - 1$	

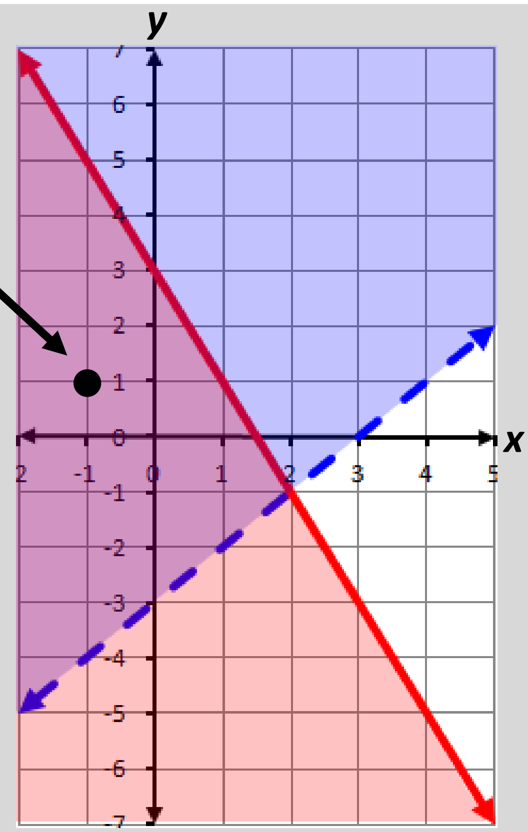
System of Linear Inequalities

Solve by graphing:

$$\begin{cases} y > x - 3 \\ y \leq -2x + 3 \end{cases}$$

The solution region contains all ordered pairs that are solutions to both inequalities in the system.

(-1,1) is one solution to the system located in the solution region.



Dependent and Independent Variable

x, independent variable
(input values or domain set)

Example:

$$y = 2x + 7$$

y, dependent variable
(output values or range set)

Dependent and Independent Variable

Determine the distance a car will travel going 55 mph.

$$d = 55h$$

independent

h	d
0	0
1	55
2	110
3	165

dependent

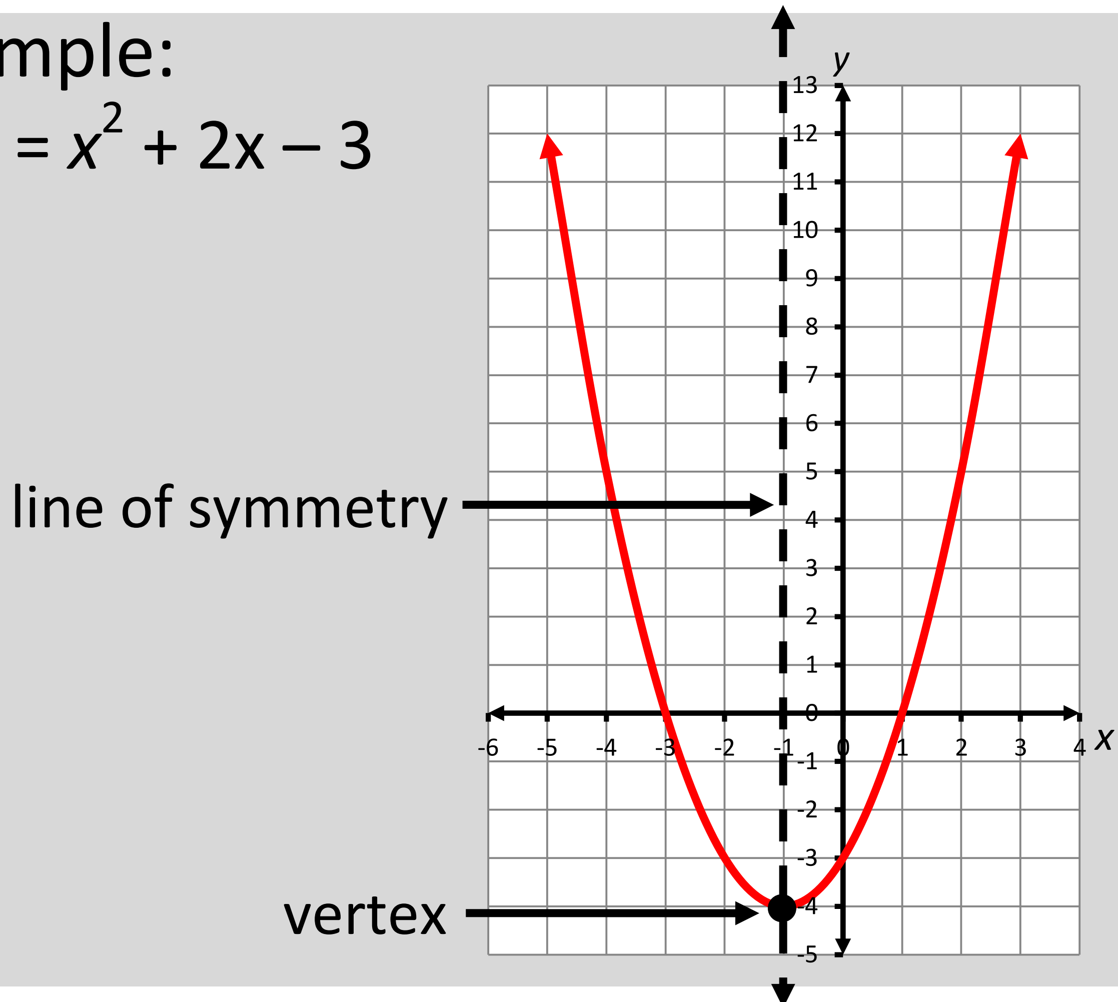
Graph of a Quadratic Equation

$$y = ax^2 + bx + c$$

$$a \neq 0$$

Example:

$$y = x^2 + 2x - 3$$



The graph of the quadratic equation is a curve (parabola) with one line of symmetry and one vertex.

Quadratic Formula

Used to find the solutions to any quadratic equation of the form, $y = ax^2 + bx + c$

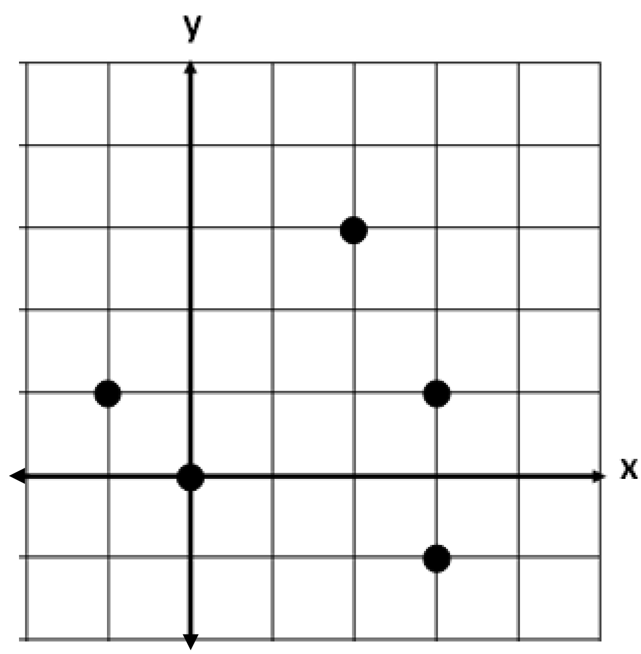
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Relations

Representations of relationships

x	*y*
-3	4
0	0
1	-6
2	2
5	-1

Example 1



Example 2

$\{(0,4), (0,3), (0,2), (0,1)\}$

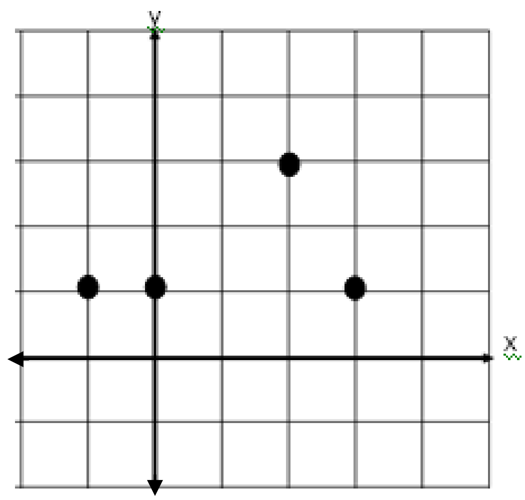
Example 3

Functions

Representations of functions

x	*y*
3	2
2	4
0	2
-1	2

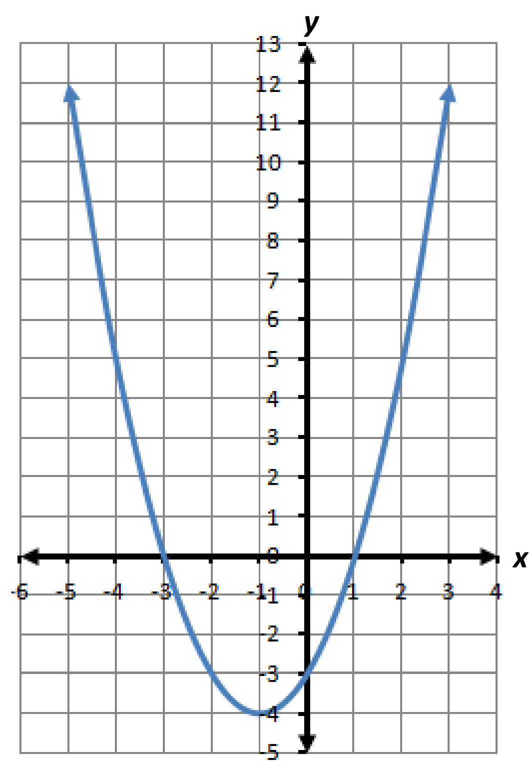
Example 1



Example 2

{(-3,4), (0,3), (1,2), (4,6)}

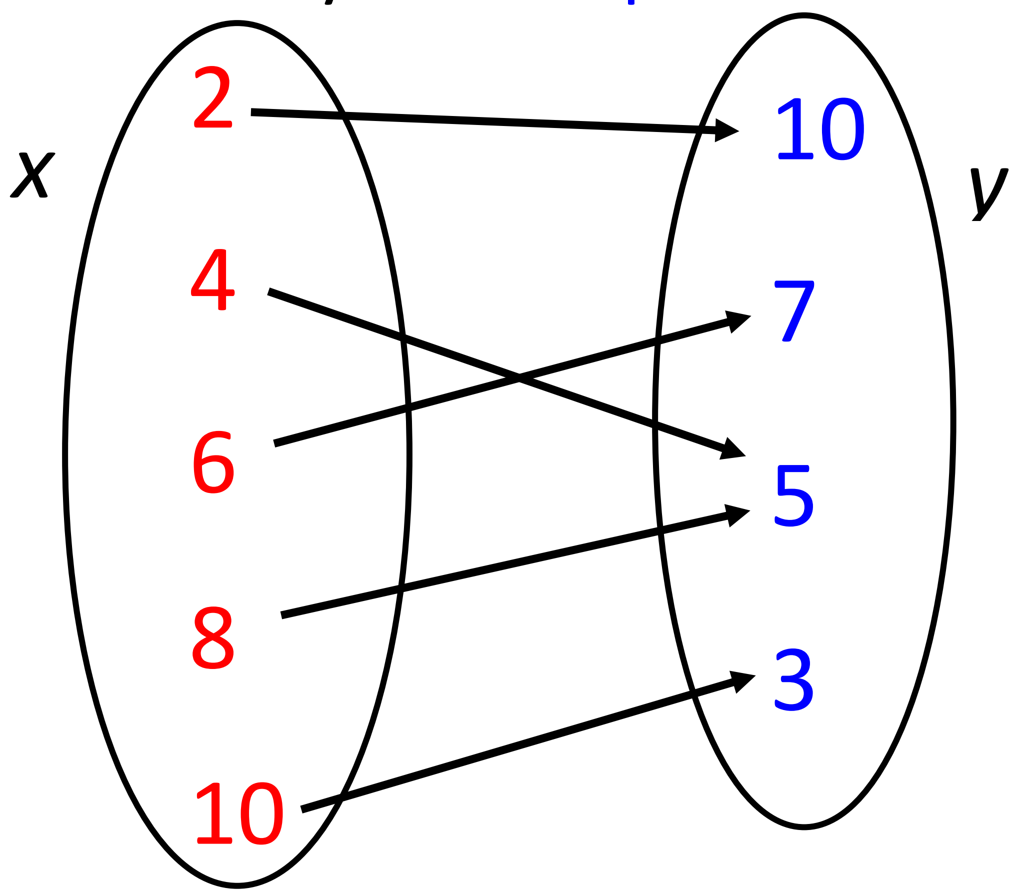
Example 3



Example 4

Function

A relationship between two quantities in which every input corresponds to exactly one output



A relation is a function if and only if each element in the domain is paired with a unique element of the range.

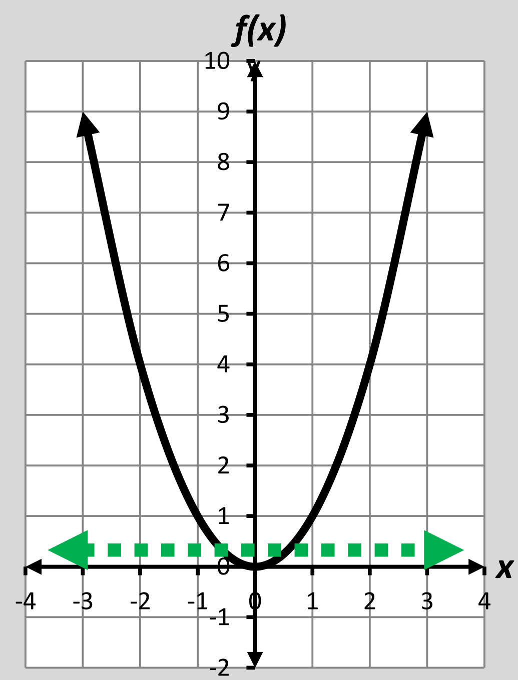
Domain

A set of input values of a relation

Examples:

input	output
x	***g*(*x*)**
-2	**0**
-1	**1**
0	**2**
1	**3**

The **domain** of *g*(*x*) is {**-2, -1, 0, 1**}.



The **domain** of *f*(*x*) is **all real numbers**.

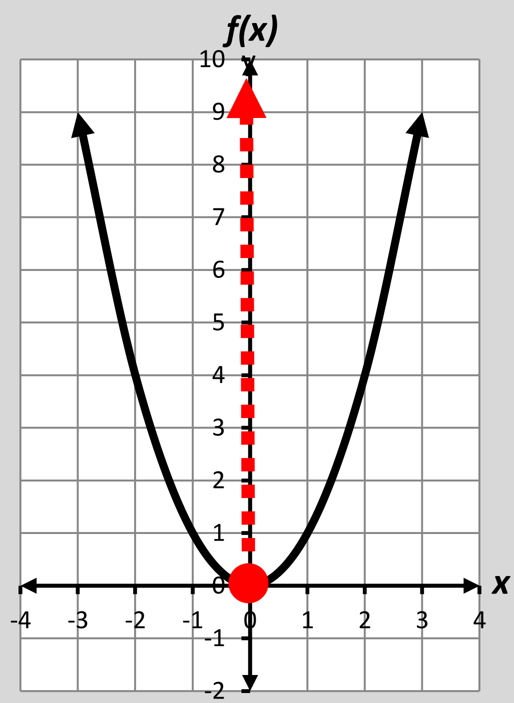
Range

A set of output values of a relation

Examples:

input	output
x	***g*(*x*)**
-2	**0**
-1	**1**
0	**2**
1	**3**

The **range** of g(x) is {**0, 1, 2, 3**}.



The **range** of *f*(*x*) is **all real numbers greater than or equal to zero**.

Function Notation

$f(x)$

$f(x)$ is read
"the value of f at x" or "f of x"

Example:

$f(x) = -3x + 5$, find $f(2)$.

$f(2) = -3(2) + 5$

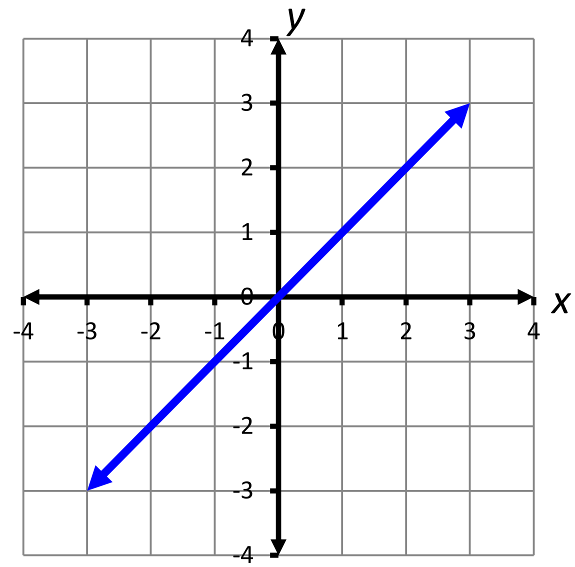
$f(2) = -6$

Letters other than f can be used to name functions, e.g., $g(x)$ and $h(x)$

Parent Functions

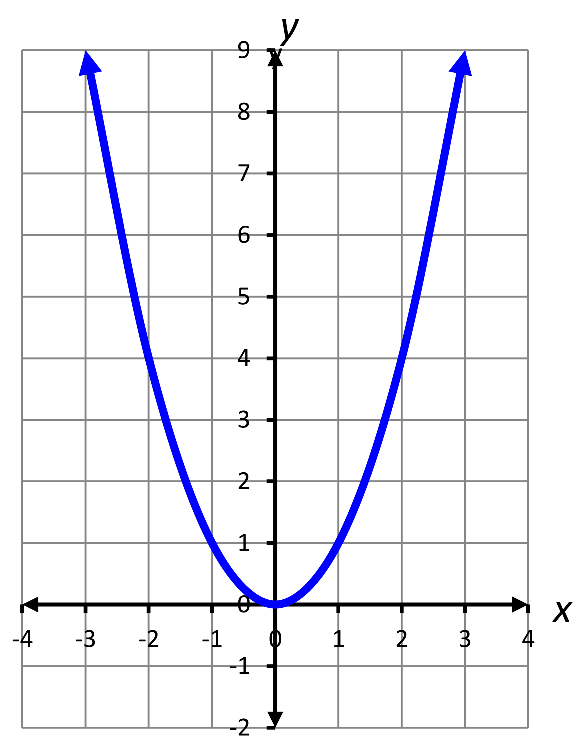
Linear

$f(x) = x$



Quadratic

$f(x) = x^2$



Transformations of Parent Functions

Parent functions can be transformed to create other members in a family of graphs.

<table>
<tr><td rowspan="4">Translations</td><td rowspan="2">$g(x) = f(x) + k$ is the graph of $f(x)$ translated vertically –</td><td>k units up when $k > 0$.</td></tr>
<tr><td>k units down when $k < 0$.</td></tr>
<tr><td rowspan="2">$g(x) = f(x - h)$ is the graph of $f(x)$ translated horizontally –</td><td>h units right when $h > 0$.</td></tr>
<tr><td>h units left when $h < 0$.</td></tr>
</table>

Transformations of Parent Functions

Parent functions can be transformed to create other members in a family of graphs.

Reflections	$\boldsymbol{g(x) = -f(x)}$ is the graph of $f(x)$ –	**reflected** over the ***x*-axis**.
	$\boldsymbol{g(x) = f(-x)}$ is the graph of $f(x)$ –	**reflected** over the ***y*-axis**.

Transformations of Parent Functions

Parent functions can be transformed to create other members in a family of graphs.

Dilations	$g(x) = a \cdot f(x)$ is the graph of $f(x)$ –	**vertical dilation** (stretch) if $a > 1$.
		vertical dilation (compression) if $0 < a < 1$.
	$g(x) = f(ax)$ is the graph of $f(x)$ –	**horizontal dilation** (compression) if $a > 1$.
		horizontal dilation (stretch) if $0 < a < 1$.

Transformational Graphing

Linear functions

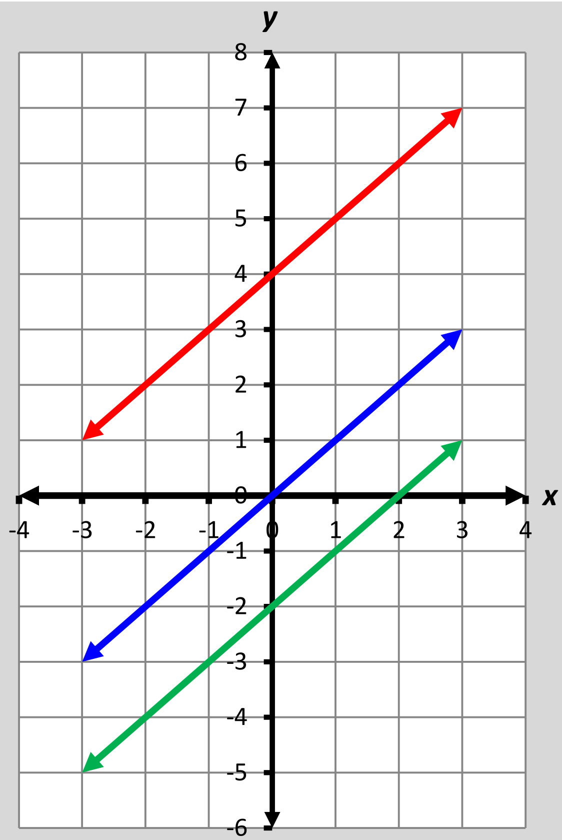
$g(x) = x + b$

Examples:

$f(x) = x$

$t(x) = x + 4$

$h(x) = x - 2$



Vertical translation of the parent function, $f(x) = x$

Transformational Graphing

Linear functions

$$g(x) = mx$$

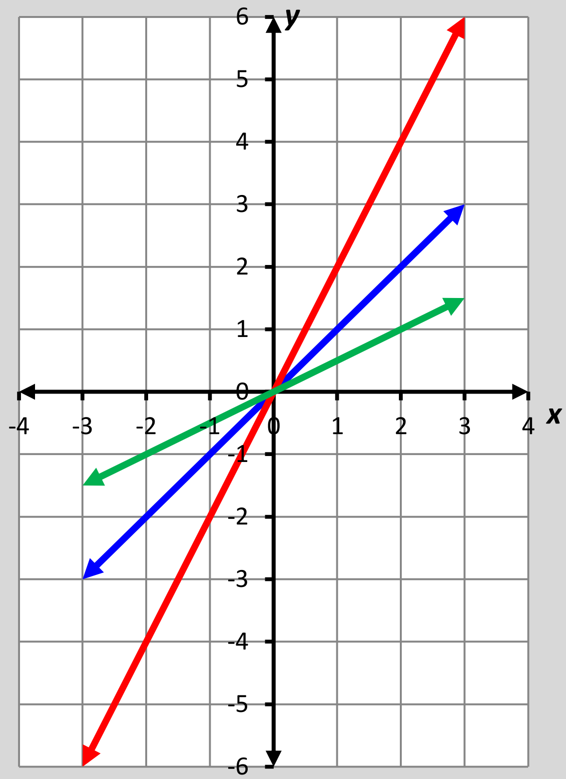
$$m>0$$

Examples:

$f(x) = x$

$t(x) = 2x$

$h(x) = \frac{1}{2}x$



Vertical dilation (stretch or compression) of the parent function, $f(x) = x$

Transformational Graphing

Linear functions

$$g(x) = mx$$

$$m < 0$$

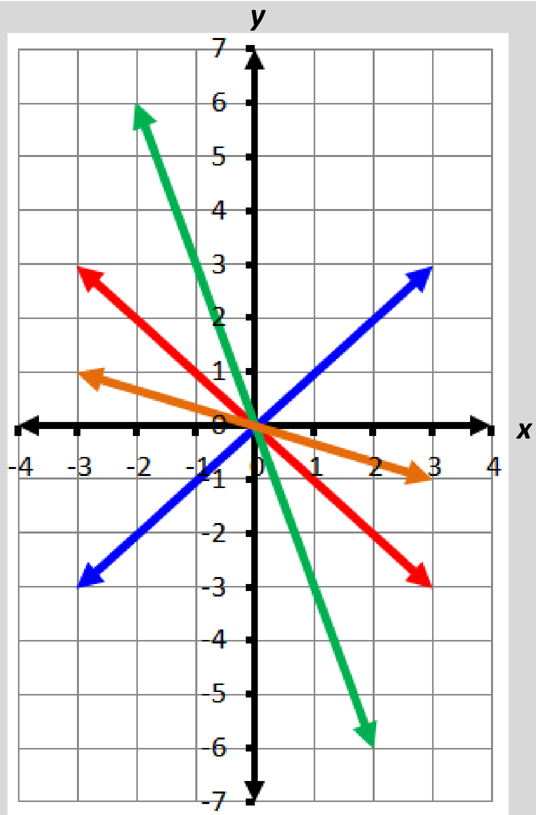
Examples:

$f(x) = x$

$t(x) = -x$

$h(x) = -3x$

$d(x) = -\frac{1}{3}x$



Vertical dilation (stretch or compression) with a reflection of $f(x) = x$

Transformational Graphing

Quadratic functions

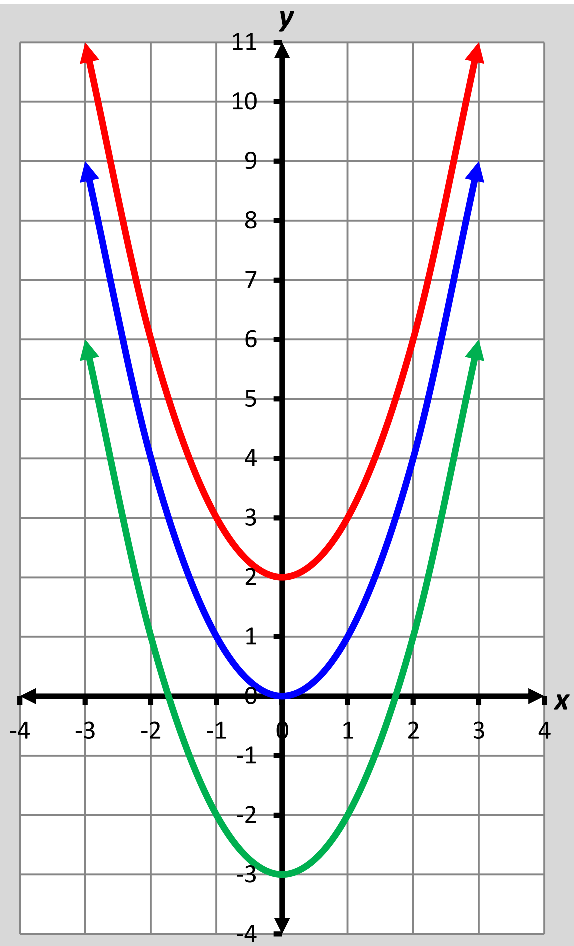
$h(x) = x^2 + c$

Examples:

$f(x) = x^2$

$g(x) = x^2 + 2$

$t(x) = x^2 - 3$



Vertical translation of $f(x) = x^2$

Transformational Graphing

Quadratic functions

$$h(x) = ax^2$$

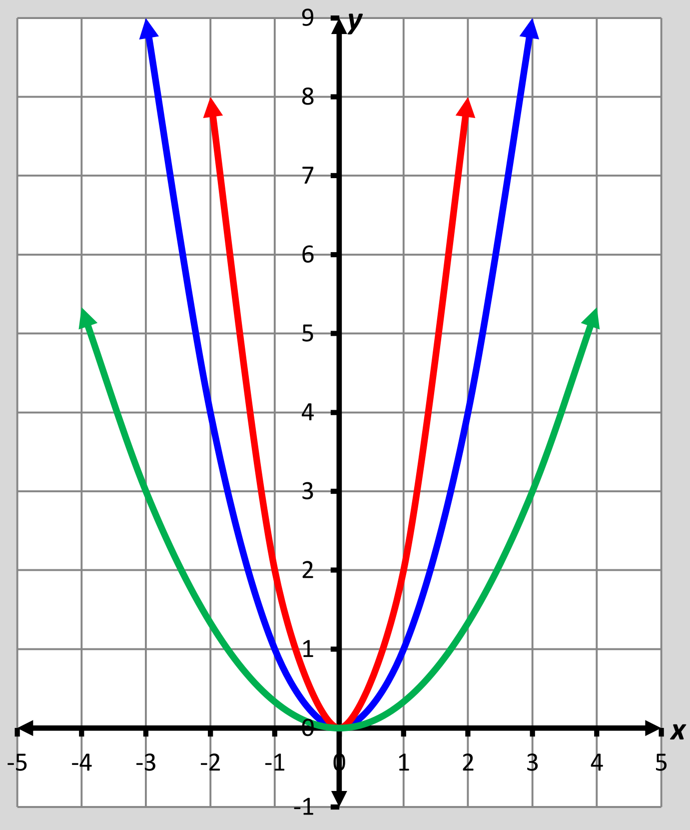
$$a > 0$$

Examples:

$f(x) = x^2$

$g(x) = 2x^2$

$t(x) = \frac{1}{3}x^2$



Vertical dilation (stretch or compression) of $f(x) = x^2$

Transformational Graphing

Quadratic functions

$$h(x) = ax^2$$

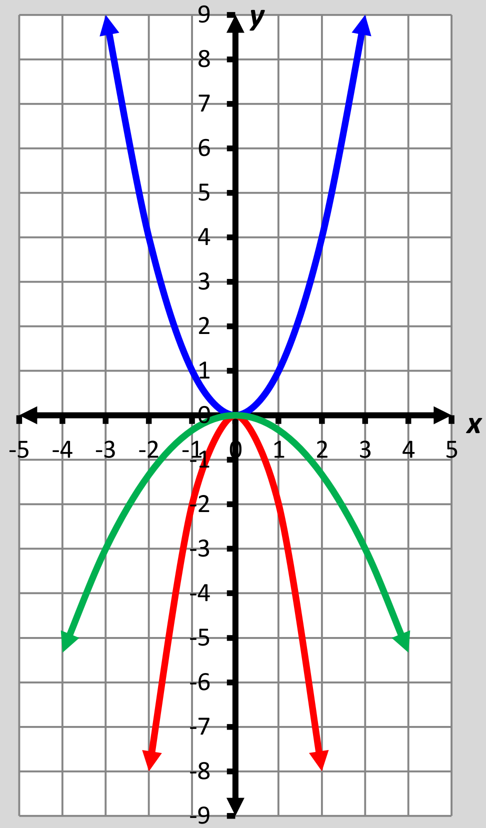
$$a < 0$$

Examples:

$f(x) = x^2$

$g(x) = -2x^2$

$t(x) = -\frac{1}{3}x^2$



Vertical dilation (stretch or compression) with a reflection of $f(x) = x^2$

Transformational Graphing

Quadratic functions

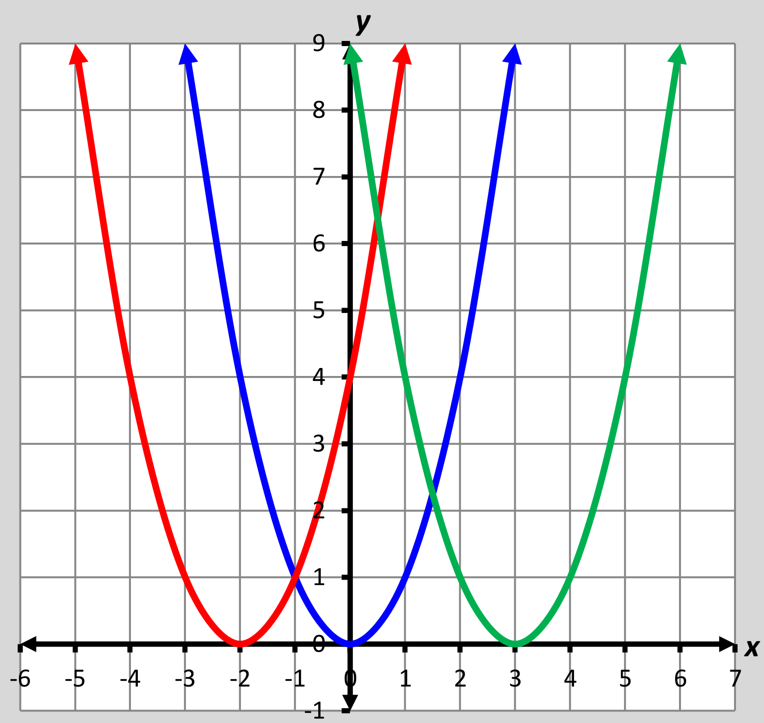
$h(x) = (x + c)^2$

Examples:

$f(x) = x^2$

$g(x) = (x + 2)^2$

$t(x) = (x - 3)^2$



Horizontal translation of $f(x) = x^2$

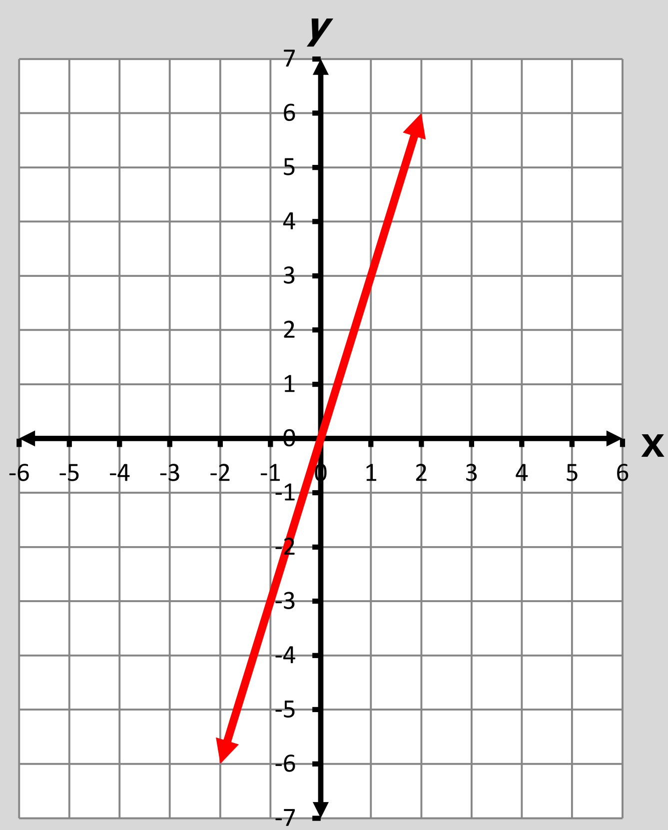
Direct Variation

$$y = \mathbf{k}x \quad \text{or} \quad \mathbf{k} = \frac{y}{x}$$

constant of variation, $k \neq 0$

Example:

$y = 3x$ or $3 = \frac{y}{x}$



The graph of all points describing a direct variation is a line passing through the origin.

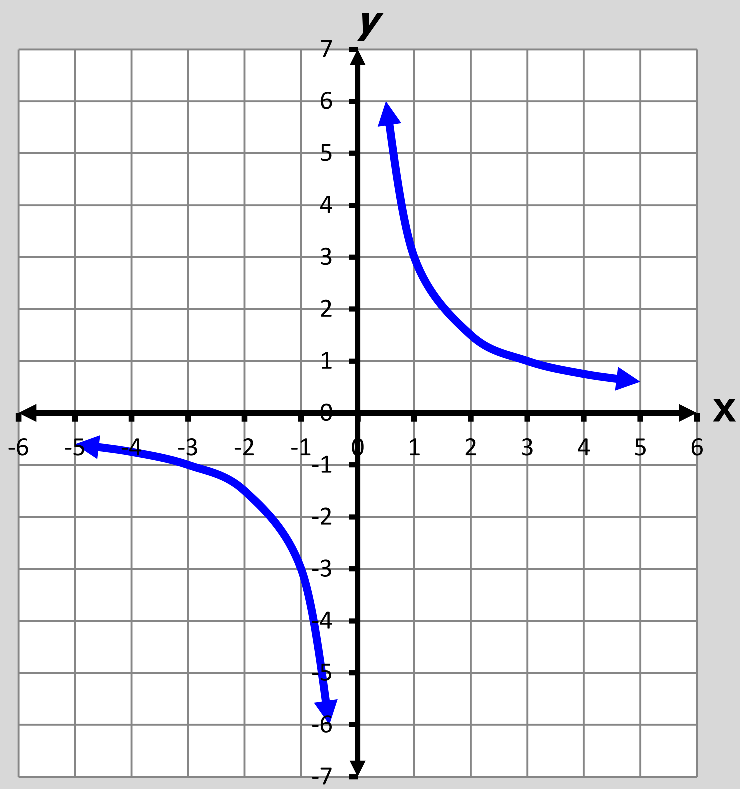
Inverse Variation

$$y = \frac{k}{x} \quad \text{or} \quad k = xy$$

constant of variation, $k \neq 0$

Example:

$$y = \frac{3}{x} \quad \text{or} \quad xy = 3$$



The graph of all points describing an inverse variation relationship are 2 curves that are reflections of each other.

Statistics Notation

$\boldsymbol{x_i}$	i^{th} element in a data set
$\boldsymbol{\mu}$	mean of the data set
$\boldsymbol{\sigma^2}$	variance of the data set
$\boldsymbol{\sigma}$	standard deviation of the data set
$\boldsymbol{n}$	number of elements in the data set

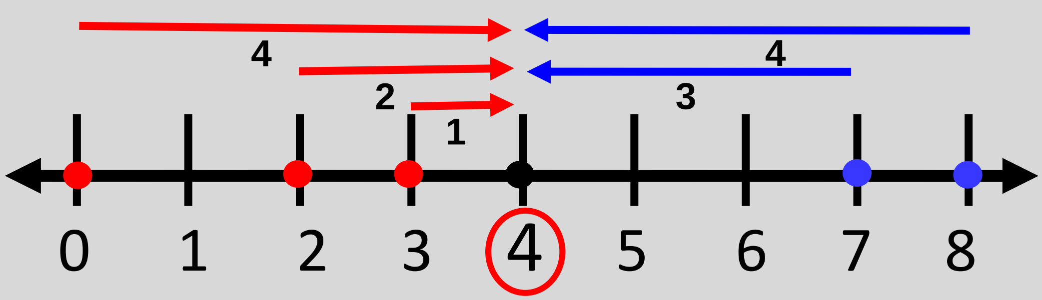
Mean

A measure of central tendency

Example: Find the mean of the given data set.

Data set: 0, 2, 3, 7, 8

Balance Point



Numerical Average

$$\mu = \frac{0+2+3+7+8}{5} = \frac{20}{5} = 4$$

Median

A measure of central tendency

Examples:

Find the median of the given data sets.

Data set: 6, 7, 8, 9, 9

The median is 8.

Data set: 5, 6, $\underbrace{8, 9}$, 11, 12

The median is 8.5.

Mode

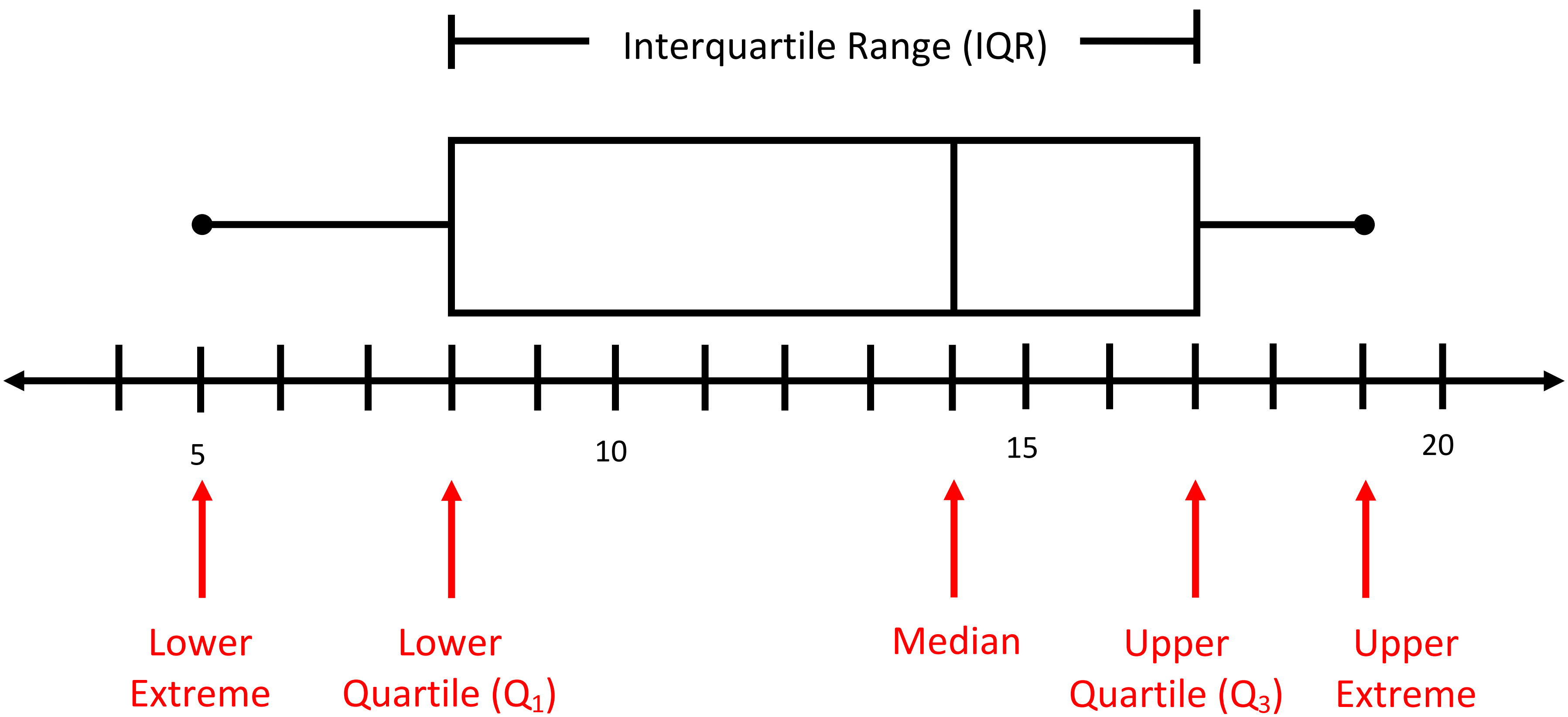
A measure of central tendency

Examples:

Data Sets	Mode
3, 4, 6, 6, 6, 6, 10, 11, 14	6
0, 3, 4, 5, 6, 7, 9, 10	none
5.2, 5.2, 5.2, 5.6, 5.8, 5.9, 6.0	5.2
1, 1, 2, 5, 6, 7, 7, 9, 11, 12	1, 7 bimodal

Box-and-Whisker Plot

A graphical representation of the five-number summary



Summation

stopping point
upper limit

summation sign

$$\sum_{i=1}^{n} x_i$$

typical element

index of
summation

starting point
lower limit

This expression means sum the values of *x*, starting at x_1 and ending at x_n.

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + \ldots + x_n$$

Example: Given the data set {3, 4, 5, 5, 10, 17}

$$\sum_{i=1}^{6} x_i = 3 + 4 + 5 + 5 + 10 + 17 = 44$$

Mean Absolute Deviation

A measure of the spread of a data set

$$\text{Mean Absolute Deviation} = \frac{\sum_{i=1}^{n} \left| x_i - \mu \right|}{n}$$

The mean of the sum of the absolute value of the differences between each element and the mean of the data set

Variance

A measure of the spread of a data set

$$\text{variance}(\sigma^2) = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}$$

The mean of the squares of the differences between each element and the mean of the data set

Standard Deviation

A measure of the spread of a data set

$$\text{standard deviation}(\sigma) = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \mu\right)^2}{n}}$$

The square root of the mean of the squares of the differences between each element and the mean of the data set or the square root of the variance

z-Score

The number of standard deviations an element is away from the mean

$$\text{z-score (z)} = \frac{x - \mu}{\sigma}$$

where x is an element of the data set, μ is the mean of the data set, and σ is the standard deviation of the data set.

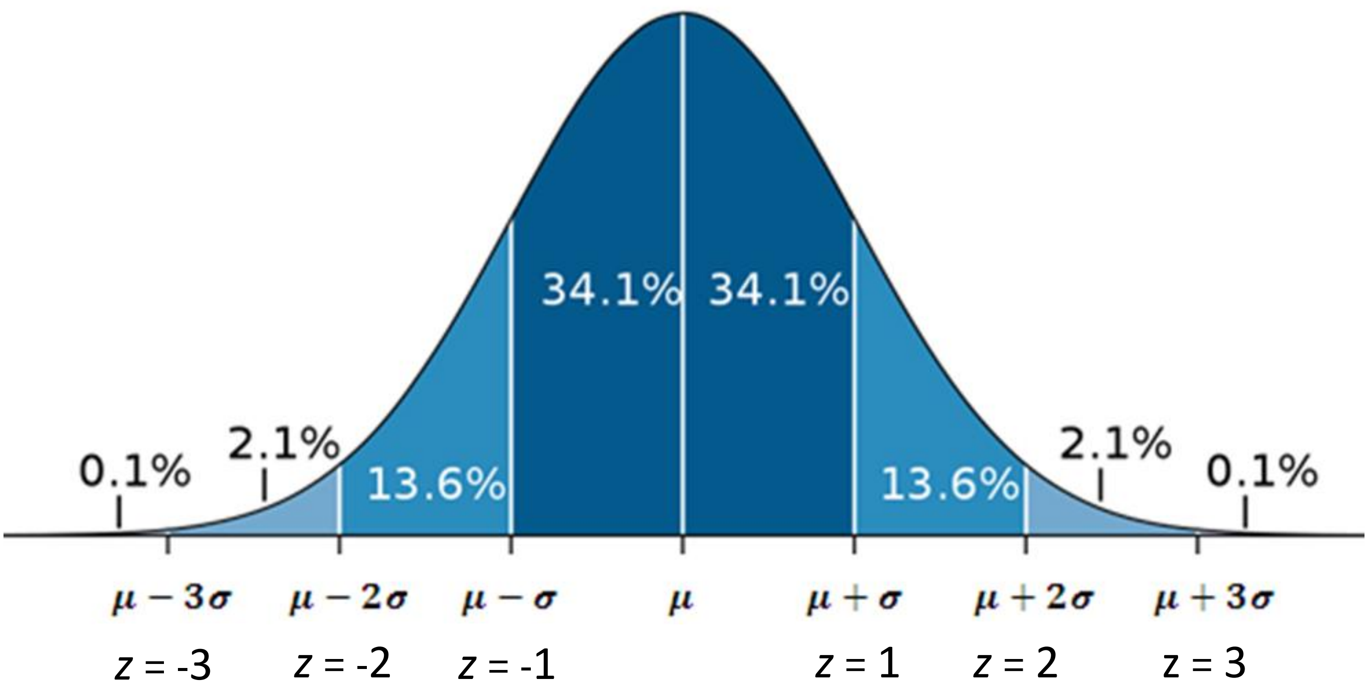
Example: Data set A has a mean of 83 and a standard deviation of 9.74. What is the z-score for the element 91 in data set A?

$$z = \frac{91-83}{9.74} = 0.821$$

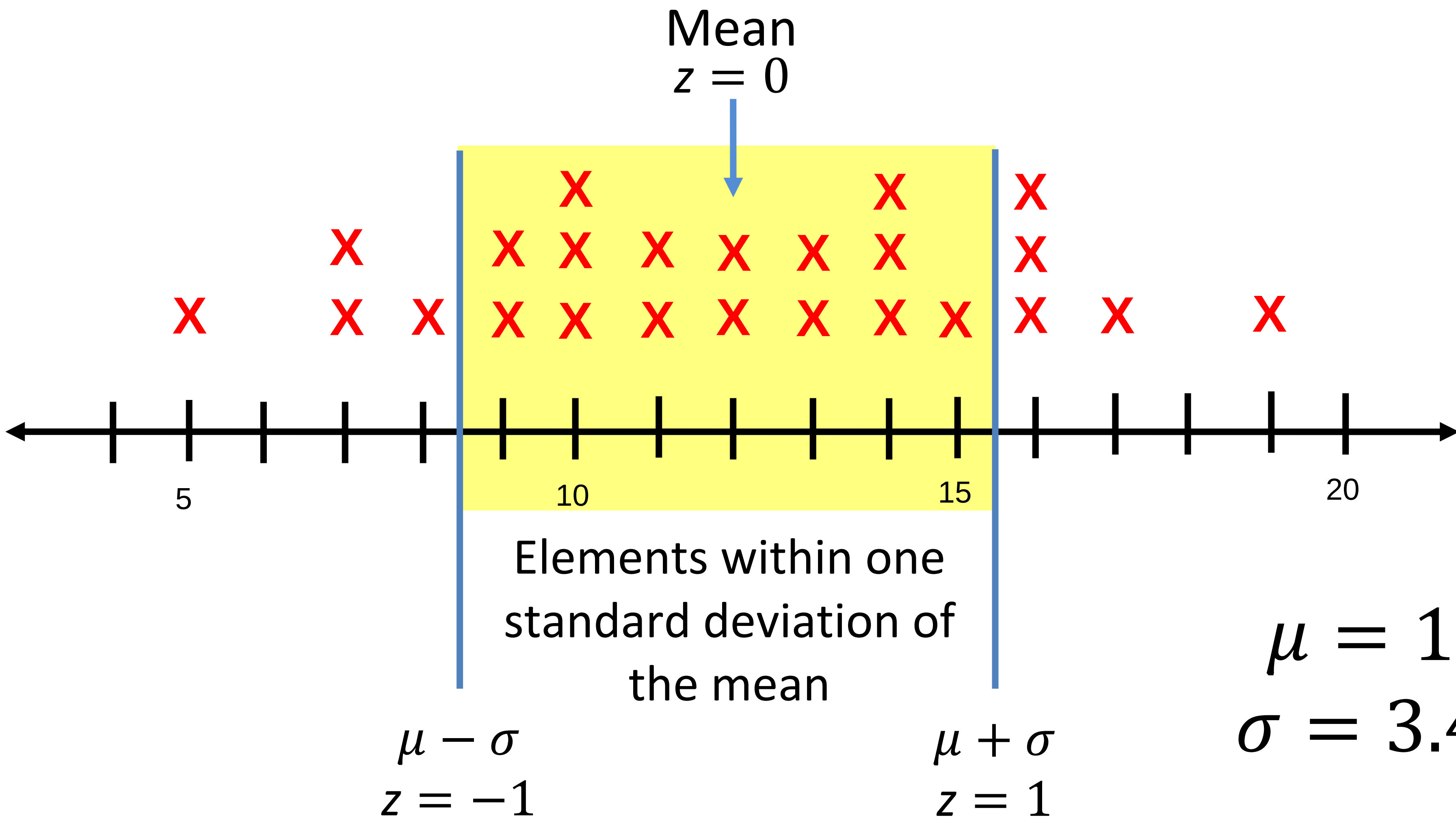
z-Score

The number of standard deviations an element is from the mean

$$\text{z-score (z)} = \frac{x - \mu}{\sigma}$$



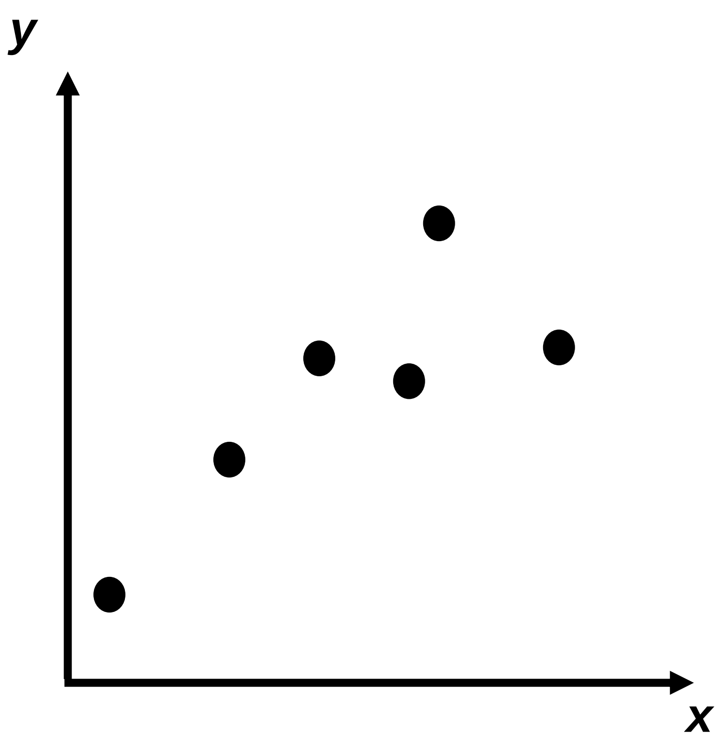
Elements within One Standard Deviation (σ)of the Mean (μ)



$\mu = 12$
$\sigma = 3.49$

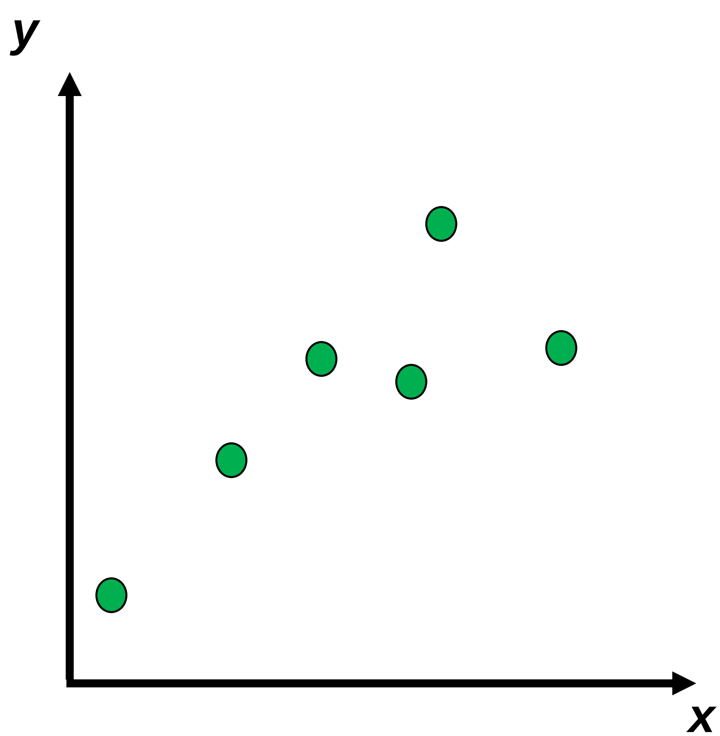
Scatterplot

Graphical representation of the relationship between two numerical sets of data



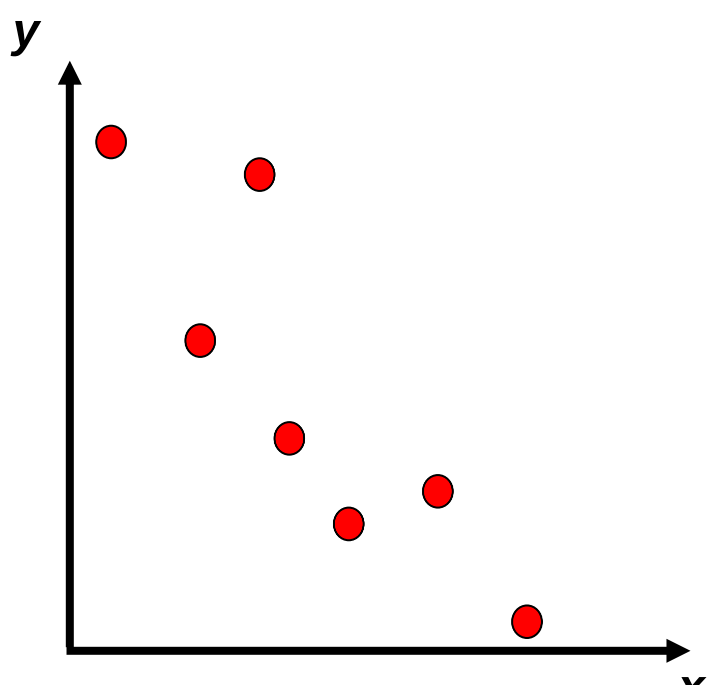
Positive Correlation

In general, a relationship where the dependent (*y*) values increase as independent values (*x*) increase



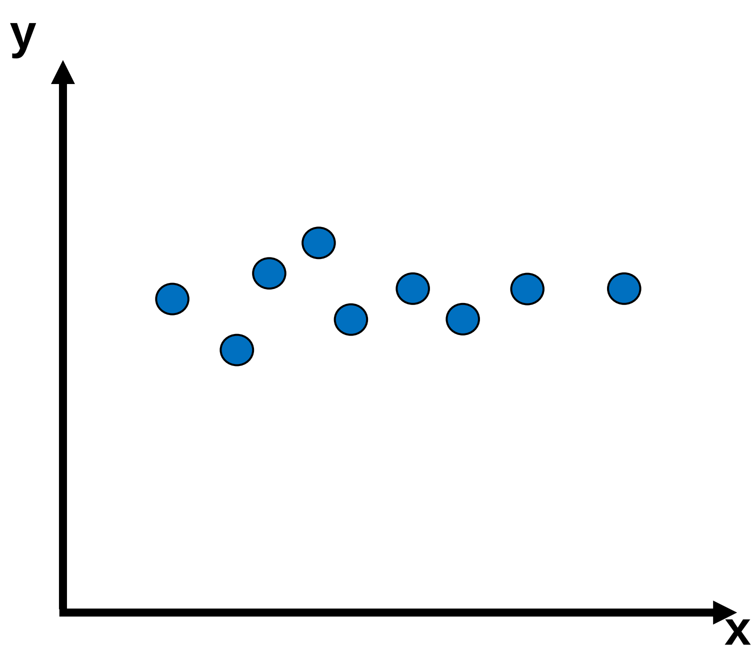
Negative Correlation

In general, a relationship where the dependent (y) values decrease as independent (x) values increase.



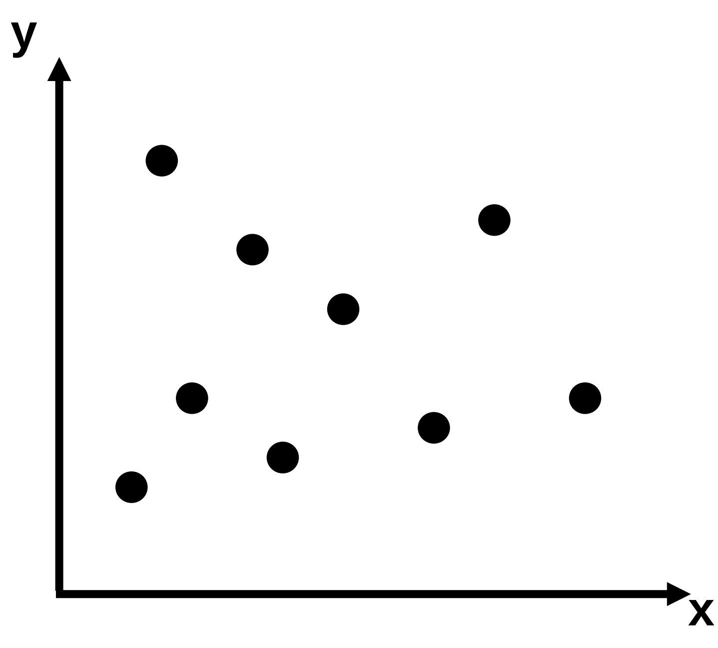
Constant Correlation

The dependent (*y*) values remain about the same as the independent (*x*) values increase.

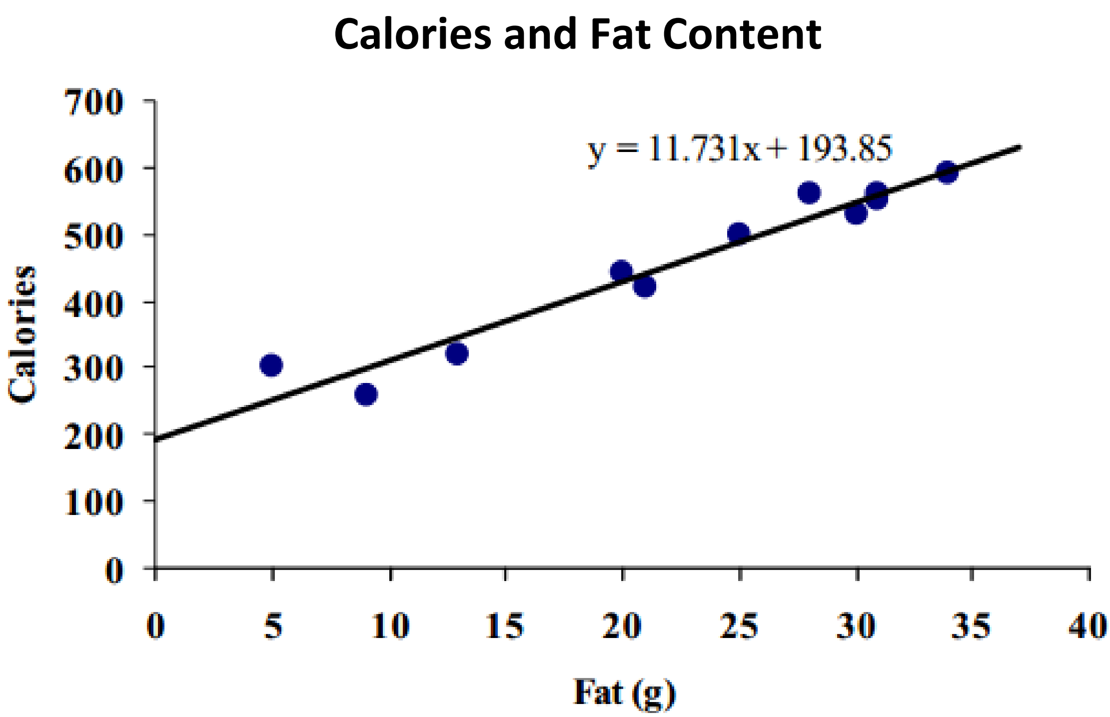


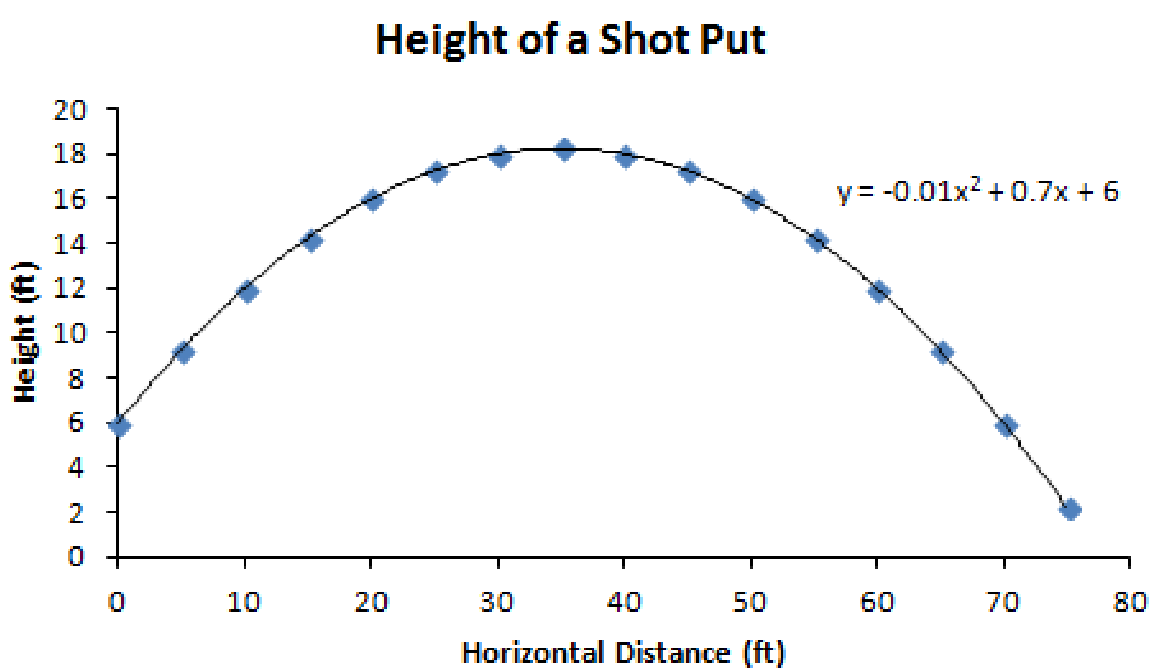
No Correlation

No relationship between the dependent (y) values and independent (x) values.



Curve of Best Fit





Outlier Data